VIII SEPOPE

VIII SIMPÓSIO DE ESPECIALISTAS EM PLANEJAMENTO DA OPERAÇÃO E EXPANSÃO ELÉTRICA

VIII SYMPOSIUM OF SPECIALISTS IN ELECTRIC OPERATIONAL AND EXPANSION PLANNING

TWO POWERFUL NETWORK MODELING APPROACHES FOR THE MODAL ANALYSIS OF HARMONIC PROBLEMS

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BACKGROUND

- Transfer Impedance (Function)

$$z_{ij}(s) = \frac{V_i(s)}{I_j(s)}$$

- System Pole

$$z_{ij}(s) = \infty \quad \text{or} \quad \frac{1}{z_{ij}(s)} = 0$$

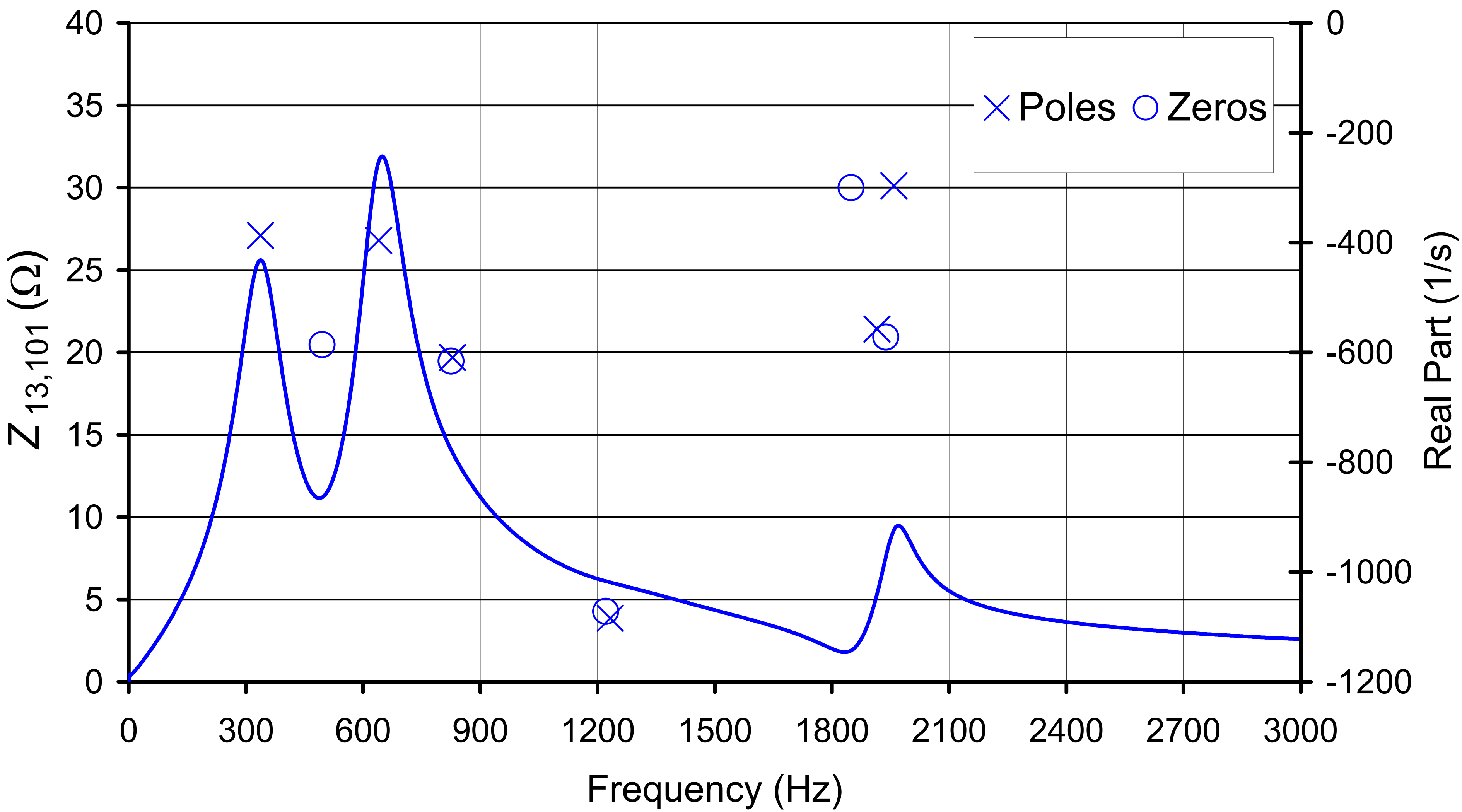
- Transfer Impedance Zero

$$z_{ij}(s) = 0$$

BACKGROUND

- If $s_k = \sigma_k + j\omega_k$ is a system pole or a transfer impedance zero then $z_{ij}(\sigma_k + j\omega_k) = \infty$ or 0, respectively, but $z_{ij}(j\omega_k) \neq \infty$ or 0.
- ω_k is a parallel resonance frequency (if s_k is a pole) or a series resonance frequency (if s_k is a zero).
- $|z_{ij}(j\omega_k)|$ has a high impedance value (very close to a local maximum) or a low impedance value (very close to a local minimum) depending on if s_k is a pole or a zero.

Background: Influency of the Pole and Zero Spectra on the Frequency Response



BACKGROUND

- Harmonic voltage performance of a system depends on the location of its poles and zeros mainly with respect to the characteristic harmonic frequencies.
- Modal analysis finds poles, zeros and their respective sensitivities to system parameters.
- Most effective parameter changes in order to reduce harmonic voltage levels.

Network Modeling Approaches

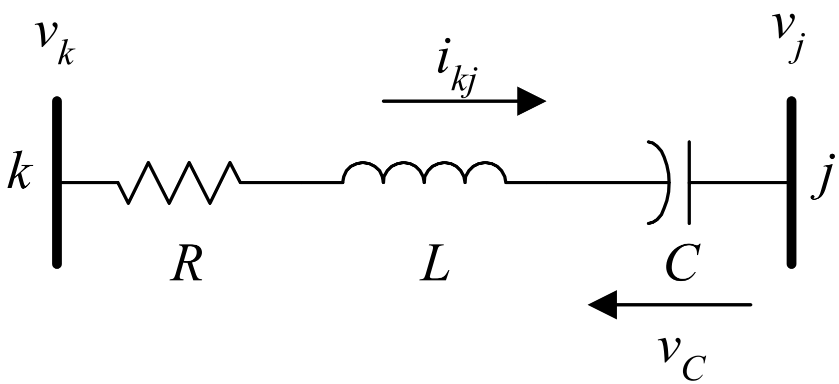
- Descriptor System and Matrix **Y**(*s*) approaches allow modal and conventional harmonic analysis of large scale networks.
- The combined advantages of the two approaches seems to be a powerful computational tool for the solutions of harmonic problems in power systems.

Descriptor System

- Main Characteristic
  - The circuit equations are written in the time-domain.
- Main Advantage
  - The system poles and the transfer function zeros (eigenvalues) can be calculated all at once using QZ factorization or one at a time using Newton based algorithm.
- Main Disadvantages
  - The modeling of frequency dependent parameters is difficult.
  - The system matrices have dimensions much larger then the number of system buses.

Descriptor System

RLC Series Branch

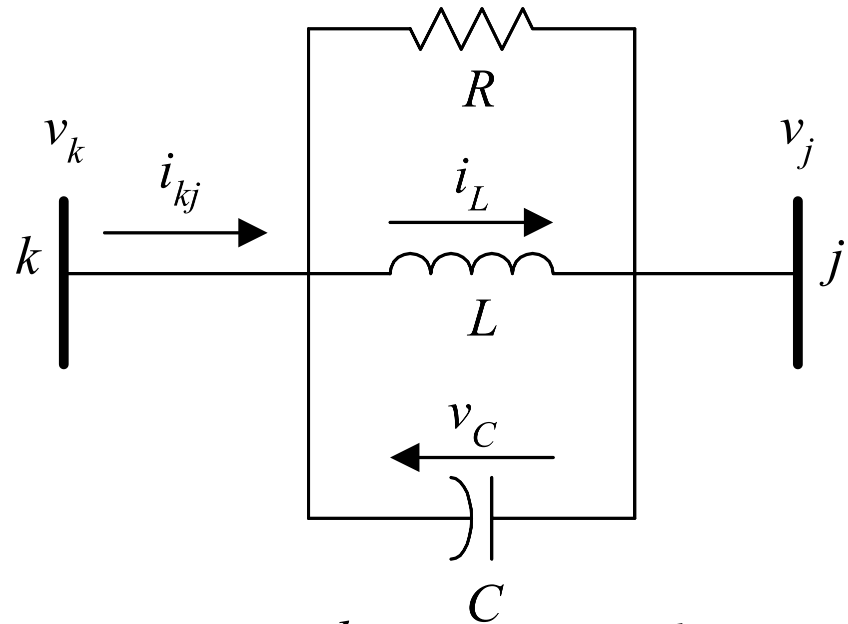


$$v_k - v_j = R\, i_{kj} + L \frac{di_{kj}}{dt} + v_C$$

$$C \frac{dv_C}{dt} = i_{kj}$$

Element variables = $[v_C\,,\, i_{kj}]$

RLC Parallel Branch



$$\frac{v_C}{R} + i_L + C \frac{dv_C}{dt} = i_{kj} \qquad L \frac{di_L}{dt} = v_C$$

$$v_C = v_k - v_j$$

Element variables = $[v_C\,,\, i_L\,,\, i_{kj}]$

Descriptor System

- KCL Equations - Kirchhoff's Current Law
  - Interconnect the various network elements

Node *k*

$$\sum_{m \in \Omega} i_{mk} = 0$$

Ω is the set of all nodes conected to node k

DESCRIPTOR SYSTEM

- The Interconnection of all system elements yields

$$\left[\begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array}\right]\left[\begin{array}{c} \dot{\mathbf{x}}_1 \\ \hline \dot{\mathbf{v}}_{nodal} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array}\right]\left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{array}\right] + \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{I} \end{array}\right]\mathbf{i}_{nodal}$$

$$\mathbf{v}_{nodal} = \left[\mathbf{0}^T \;\middle|\; \mathbf{I}\right]\left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{array}\right]$$

$\mathbf{x}_1 \rightarrow$ Vector of all element variables

- Compact form of the Descriptor System Equations

$$\mathbf{T}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$

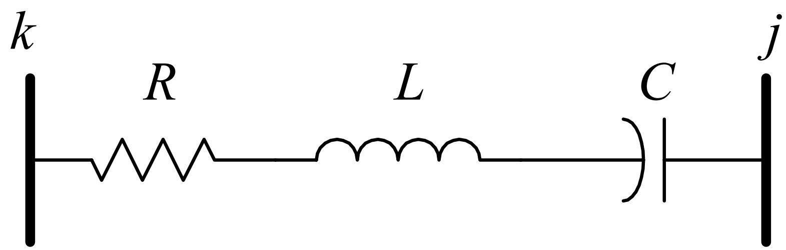
$$\mathbf{y} = \mathbf{C}\,\mathbf{x}$$

MATRIX Y(*s*)

- Main Characteristic
  - Circuit equations are written in the s-domain.
- Main Advantages
  - Modeling of frequency dependent parameters is very easy.
  - System matrices (**Y**(*s*) and its derivative with respect to *s*) have dimensions equal to the number of system buses.
- Main Disadvantage
  - System poles and the transfer function zeros (eigenvalues) can only be calculated one at a time.

MATRIX Y(s)

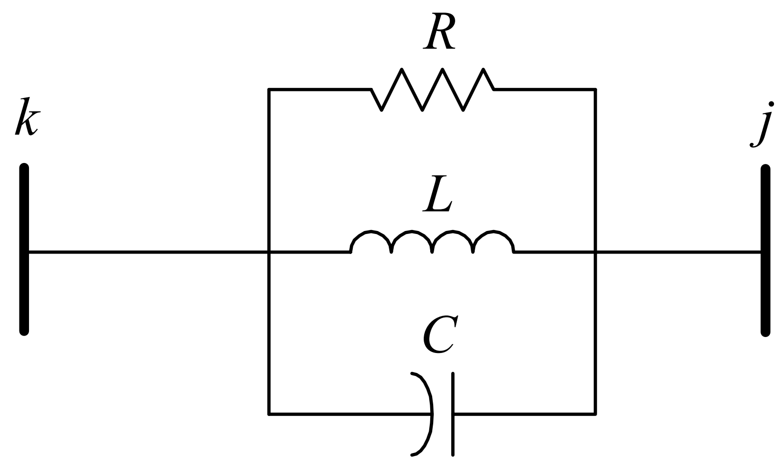
RLC Series Branch



$$y_{series} = \frac{sC}{s^2LC + sRC + 1}$$

$$\frac{dy_{series}}{ds} = -\frac{\left(s^2LC - 1\right)}{Cs^2}\, y_{series}^2$$

RLC Parallel Branch



$$y_{parallel} = \frac{1}{R} + \frac{1}{sL} + sC$$

$$\frac{dy_{parallel}}{ds} = C - \frac{1}{s^2L}$$

MATRIX Y(*s*)

- A diagonal element y_{ii} of **Y**(*s*) is equal to the summation of all elementary admittances connected to node *i*.
- An off-diagonal element y_{ij} of **Y**(*s*) is equal to the negative of the summation of all elementary admittances connected between the nodes *i* and *j*.
- The derivative matrix of **Y**(*s*) is built using the same rules used to build **Y**(*s*) but using the derivatives of the elementary admittances.

Dominant Pole Algorithm

Harmonic Impedance

$$z_{ij}(s) = V_i / I_j = \mathbf{c}^t \ \mathbf{Y}^{-1}(s)\mathbf{b}$$

System Poles

$$1/z_{ij}(s) = 0$$

Newton-Raphson

$$s^{(k+1)} = s^{(k)} + \left(\frac{dz_{ij}}{ds}\right)^{-1} z_{ij}$$

Harmonic Impedance Derivative

$$\frac{dz_{ij}(s)}{ds} = -\mathbf{w}^t \frac{d\mathbf{Y}(s)}{ds}\mathbf{v}$$

Auxiliary Equations

$$\mathbf{Y}(s)\,\mathbf{v} = \mathbf{b} \ ; \ \mathbf{Y}(s)^t \ \mathbf{w} = \mathbf{c}$$

POLE AND ZERO SENSITIVITIES TO A SYSTEM PARAMETER

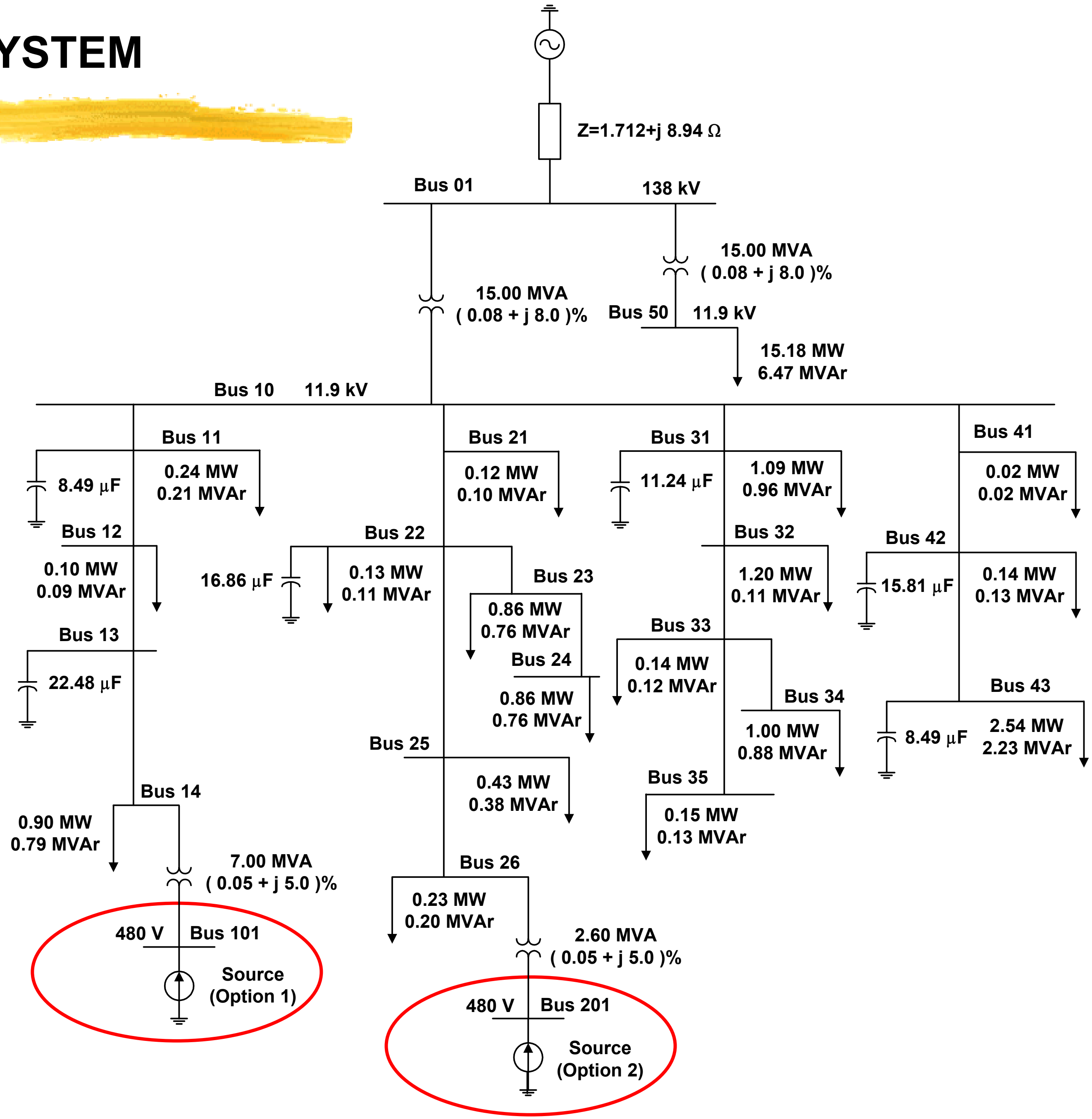
The **Y** matrix is considered as a function of the complex frequency *s* and a system parameter *p*.

Sensitivity Equation

$$\frac{ds_k}{dp_l} = \frac{\mathbf{w}^t \dfrac{\partial \mathbf{Y}(s,p)}{\partial s} \mathbf{v}}{\mathbf{w}^t \dfrac{\partial \mathbf{Y}(s,p)}{\partial p} \mathbf{v}} \Bigg|_{\substack{s = s_k \\ p = p_l}}$$

The imaginary part of a pole (or zero) sensitivity is the rate of change of the associate parallel (or series) resonance frequency with respect to a system parameter.

TEST SYSTEM

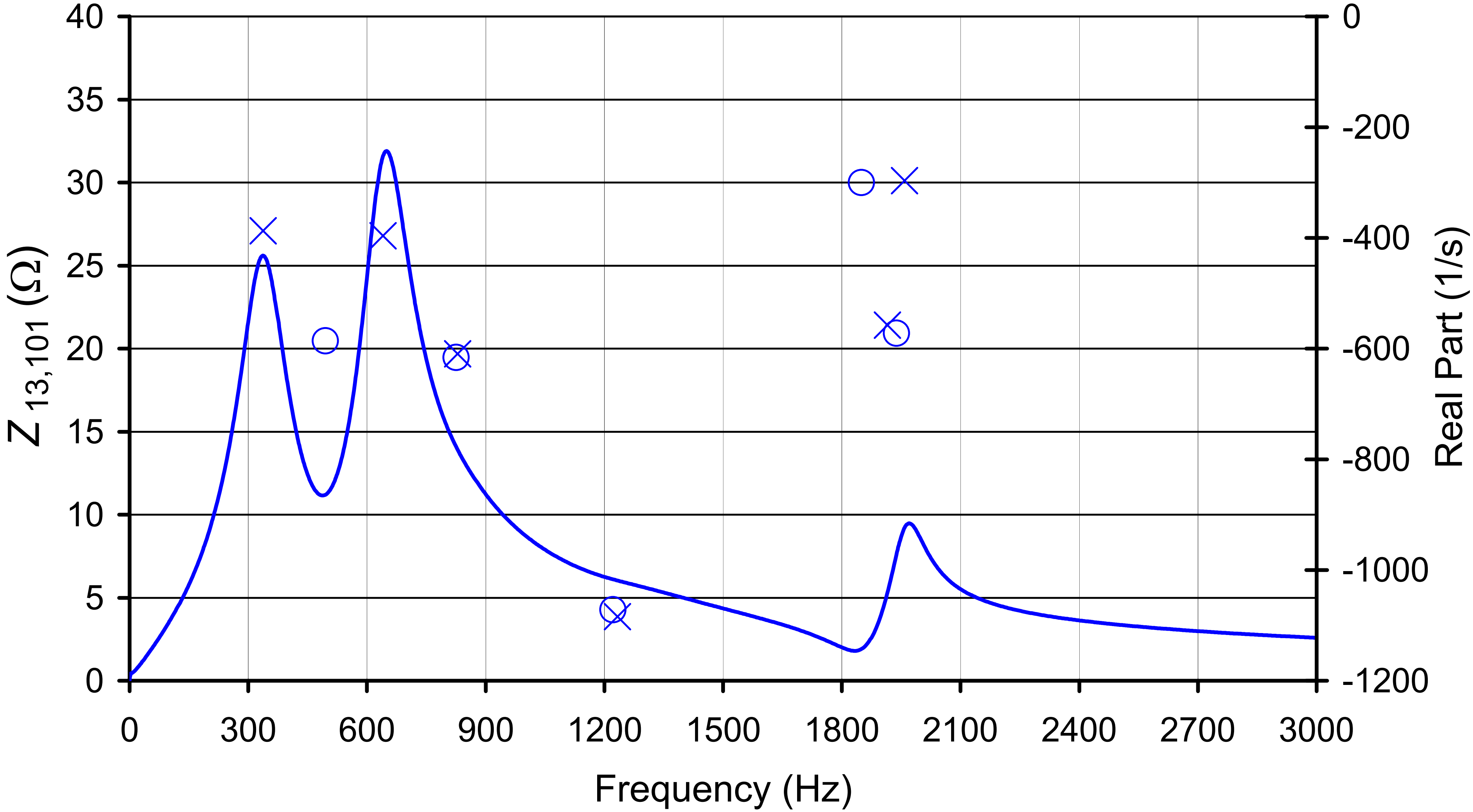

Z=1.712+j 8.94 Ω
Bus 01
138 kV
15.00 MVA
(0.08 + j 8.0)%
Bus 50
11.9 kV
15.18 MW
6.47 MVAr
15.00 MVA
(0.08 + j 8.0)%
Bus 10
11.9 kV
Bus 11
8.49 μF
0.24 MW
0.21 MVAr
Bus 12
0.10 MW
0.09 MVAr
Bus 13
22.48 μF
Bus 14
0.90 MW
0.79 MVAr
7.00 MVA
(0.05 + j 5.0)%
480 V
Bus 101
Source
(Option 1)
Bus 21
0.12 MW
0.10 MVAr
Bus 22
16.86 μF
0.13 MW
0.11 MVAr
Bus 23
0.86 MW
0.76 MVAr
Bus 24
0.86 MW
0.76 MVAr
Bus 25
0.43 MW
0.38 MVAr
Bus 26
0.23 MW
0.20 MVAr
2.60 MVA
(0.05 + j 5.0)%
480 V
Bus 201
Source
(Option 2)
Bus 31
11.24 μF
1.09 MW
0.96 MVAr
Bus 32
1.20 MW
0.11 MVAr
Bus 33
0.14 MW
0.12 MVAr
Bus 34
1.00 MW
0.88 MVAr
Bus 35
0.15 MW
0.13 MVAr
Bus 41
0.02 MW
0.02 MVAr
Bus 42
15.81 μF
0.14 MW
0.13 MVAr
Bus 43
8.49 μF
2.54 MW
2.23 MVAr

HARMONIC PROBLEM

- Which is the best bus for placing a 12-pulse industrial rectifier: Bus 101 (Option 1) or Bus 201 (Option 2)
  - Total Current at Fundamental Frequency: 5.5 kA
  - Harmonic Current Components:

h	11	13	23	25
f(Hz)	660	780	1380	1500
I_h(%)	9.0	8.0	4.0	4.0

OPTION 1: INFLUENCY OF THE POLE AND ZERO SPECTRA ON THE FREQUENCY RESPONSE



Option 1: Pole Sensitivities

Frequency Associated with the Poles and Their Sensitivities (Hz/μF) to Capacitor Changes

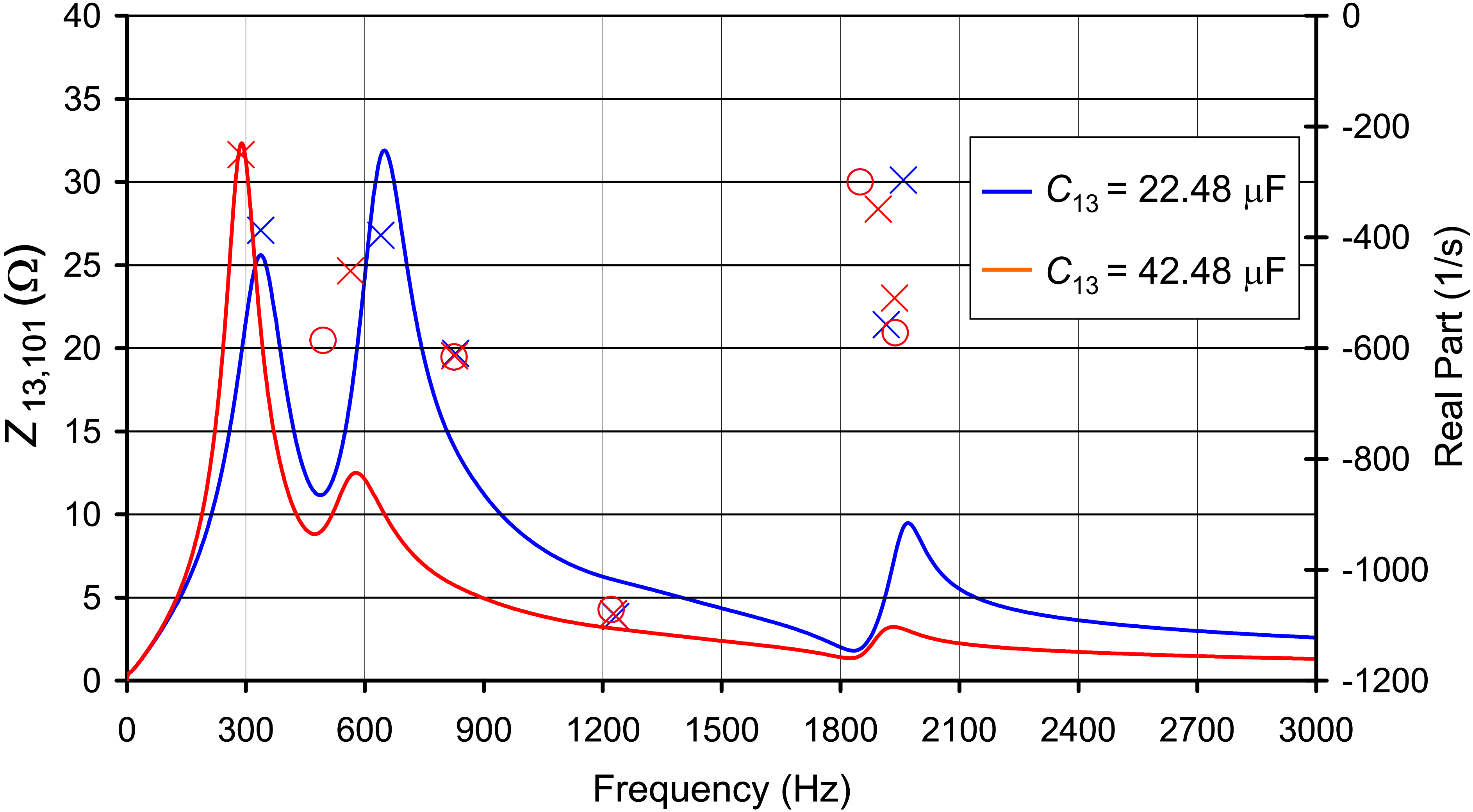
	P_1	P_2	P_3	P_4	P_5	P_6
f (Hz)→	337	640	829	1233	1916	1959
C_{11}	-1.936	-0.903	0.004	-0.697	0.836	-99.888
C_{13}	-2.731	-7.048	-0.242	-0.541	-0.170	-3.978
C_{22}	-1.682	-1.635	-16.108	-3.294	-0.124	-0.083
C_{31}	-1.387	-0.571	-0.527	-46.357	-1.701	-2.152
C_{42}	-1.815	-3.978	-3.890	-0.286	-25.237	-3.511
C_{43}	-2.036	-6.260	-8.375	-2.971	-63.705	5.768

Option 1: Zero Sensitivities

Frequency Associated with the Zeros of $Z_{101,13}$ and Their Sensitivities (Hz/μF) to Capacitor Changes

	Z_1	Z_2	Z_3	Z_4	Z_5
f (Hz)→	495	825	1221	1849	1938
C_{11}	-0.659	-0.109	-1.981	-99.619	-3.721
C_{13}	0.000	0.000	0.000	0.000	0.000
C_{22}	-4.121	-14.941	-3.814	-0.242	0.004
C_{31}	-2.588	-0.550	-44.867	-5.162	0.200
C_{42}	-5.207	-4.578	-0.344	0.918	-29.679
C_{43}	-6.786	-9.716	-3.307	-4.331	-53.585

OPTION 1: POLE SHIFT EFFECTS

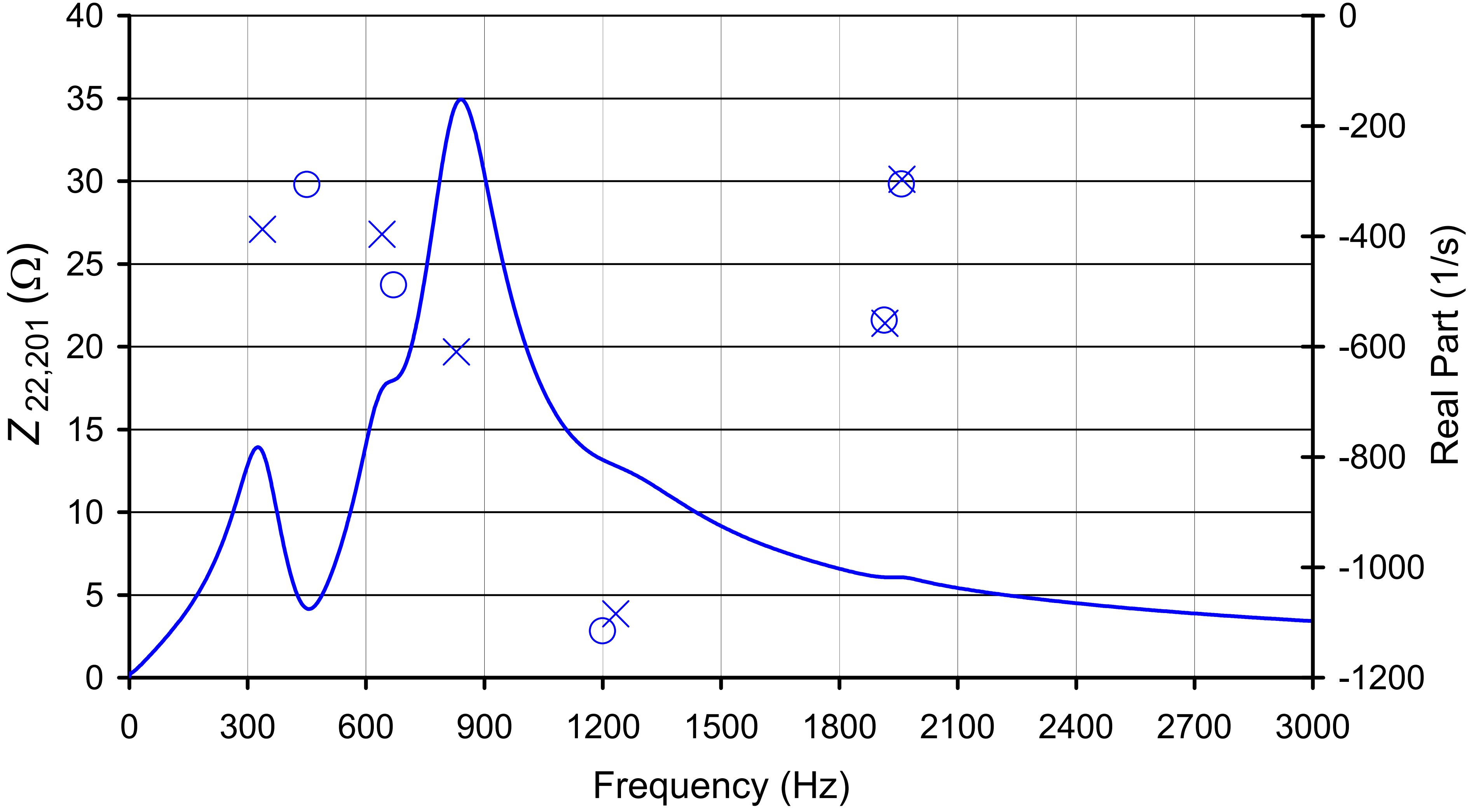


Option 1: Proposed Solution

Harmonic Voltage Distortion
Spectrum at Bus 13

Harmonic Order	Original System (%)	Proposed Solution (%)	Limits (%)
11	9.15	2.78	3.5
13	4.33	1.65	3.0
23	0.662	0.350	1.5
25	0.563	0.309	1.5

Option 2: Influency of the Pole and Zero Spectra on the Frequency Response



Option 2: Pole Sensitivities

Frequency Associated with the Poles and Their Sensitivities (Hz/μF) to Capacitor Changes

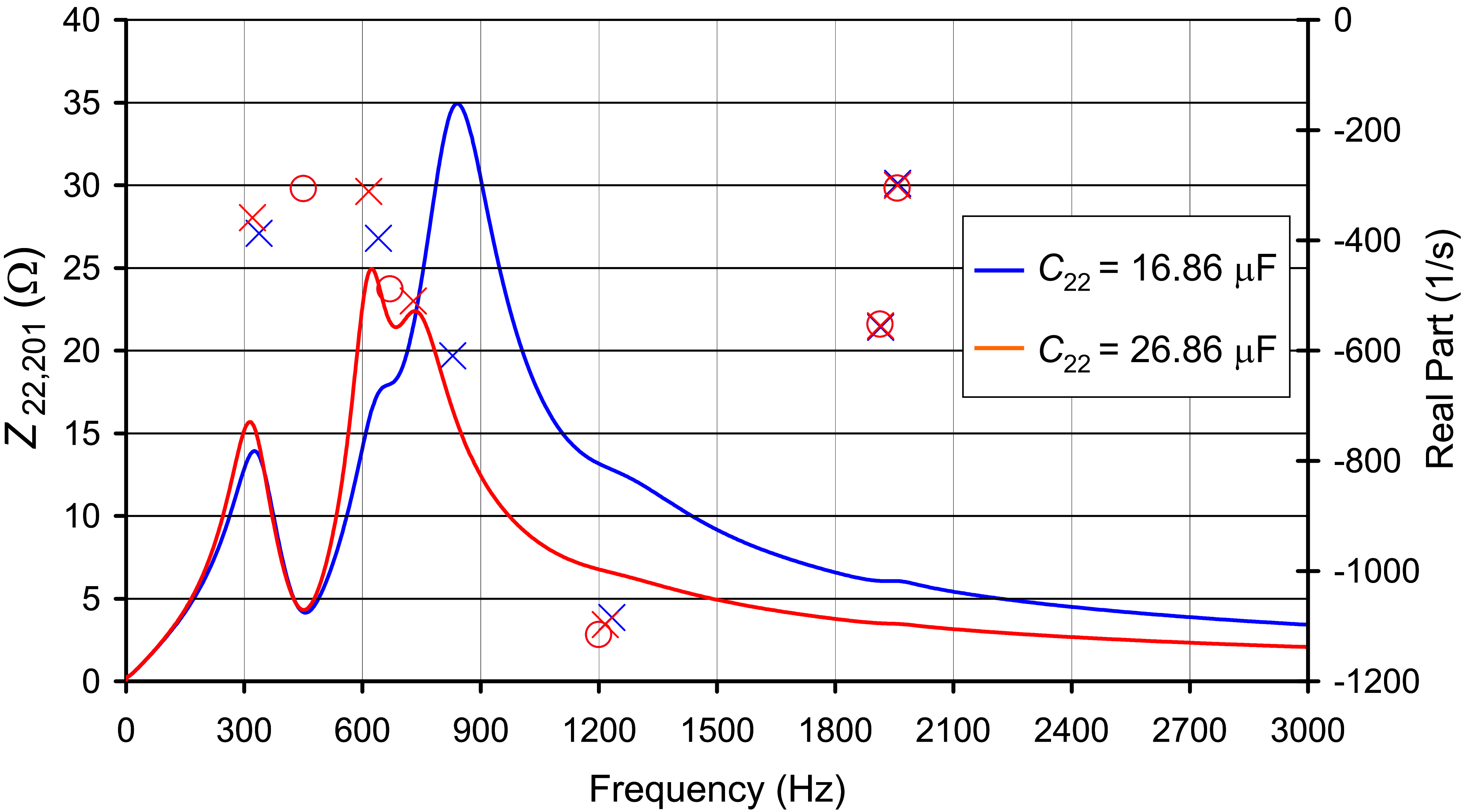
	P_1	P_2	P_3	P_4	P_5	P_6
f (Hz)→	337	640	829	1233	1916	1959
C_{11}	-1.936	-0.903	0.004	-0.697	0.836	-99.888
C_{13}	-2.731	-7.048	-0.242	-0.541	-0.170	-3.978
C_{22}	-1.682	-1.635	-16.108	-3.294	-0.124	-0.083
C_{31}	-1.387	-0.571	-0.527	-46.357	-1.701	-2.152
C_{42}	-1.815	-3.978	-3.890	-0.286	-25.237	-3.511
C_{43}	-2.036	-6.260	-8.375	-2.971	-63.705	5.768

OPTION 2: ZERO SENSITIVITIES

Frequency Associated with the Zeros of $Z_{201,22}$ and Their Sensitivities (Hz/μF) to Capacitor Changes

	Z_1	Z_2	Z_3	Z_4	Z_5
f (Hz)→	450	670	1199	1914	1958
C_{11}	-3.018	-0.269	-0.900	-0.700	-98.199
C_{13}	-6.258	-3.683	-0.798	-0.257	-3.908
C_{22}	0.000	0.000	0.000	0.000	0.000
C_{31}	-1.063	-0.735	-47.545	-1.959	-2.180
C_{42}	-1.892	-7.970	-0.552	-24.396	-4.303
C_{43}	-2.227	-13.092	-4.552	-63.444	5.446

OPTION 2: POLE SHIFT EFFECTS



Option 2: Pole Sensitivities

Frequency Associated with the Poles and Their Sensitivities (Hz/μF) to Capacitor Changes for C_{22}=26.86 μF

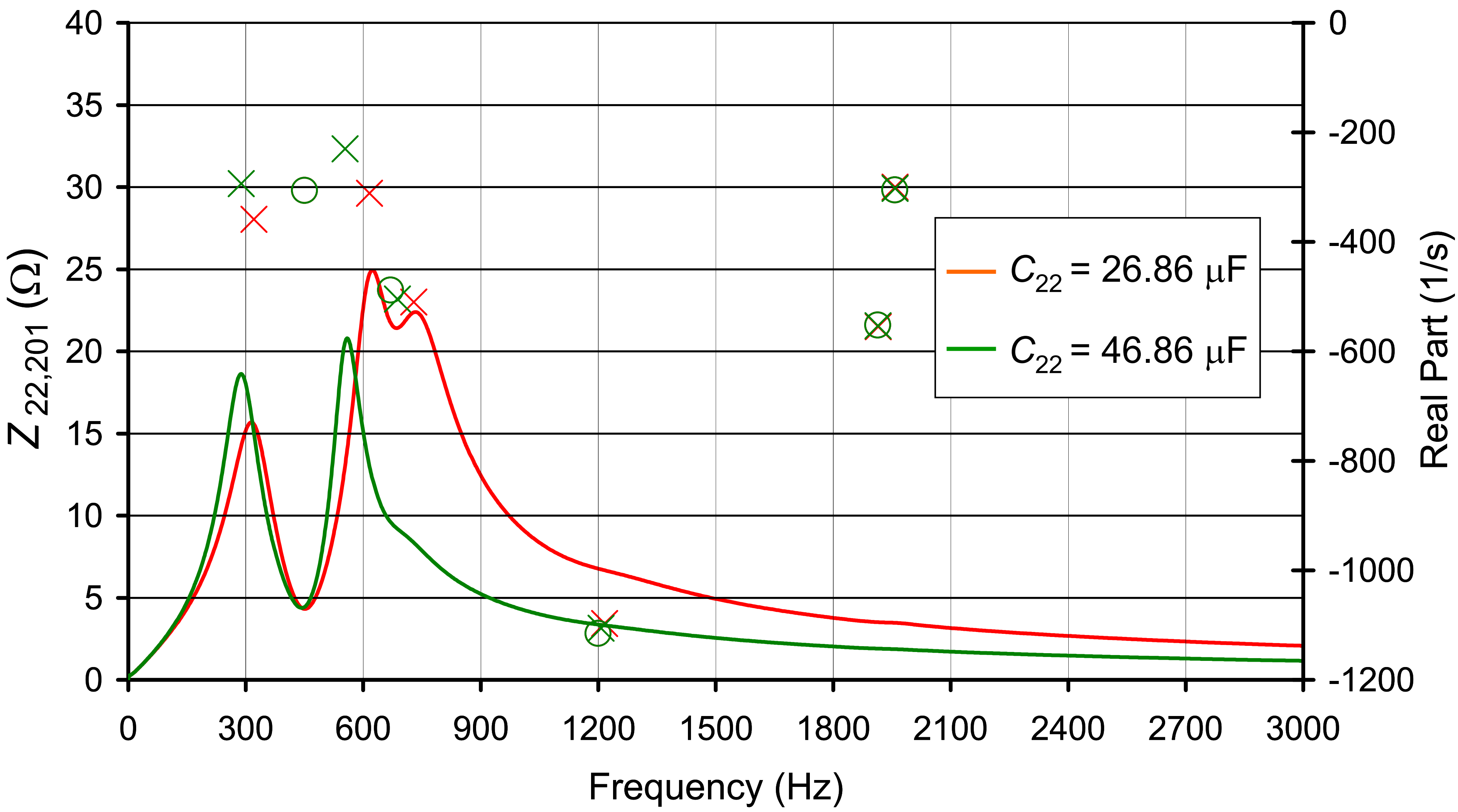
	P_1	P_2	P_3	P_4	P_5	P_6
f (Hz)→	321	616	729	1216	1915	1958
C_{11}	-1.591	-1.284	-0.007	-0.808	0.312	-99.316
C_{13}	-2.175	-7.587	-0.151	-0.668	-0.201	-3.955
C_{22}	-1.677	-3.275	-5.358	-0.845	-0.044	-0.024
C_{31}	-1.185	-0.240	-0.060	-47.422	-1.799	-2.172
C_{42}	-1.505	-1.213	-6.941	-0.406	-24.920	-3.808
C_{43}	-1.669	-1.789	-12.465	-3.737	-63.681	5.715

OPTION 2: ZERO SENSITIVITIES

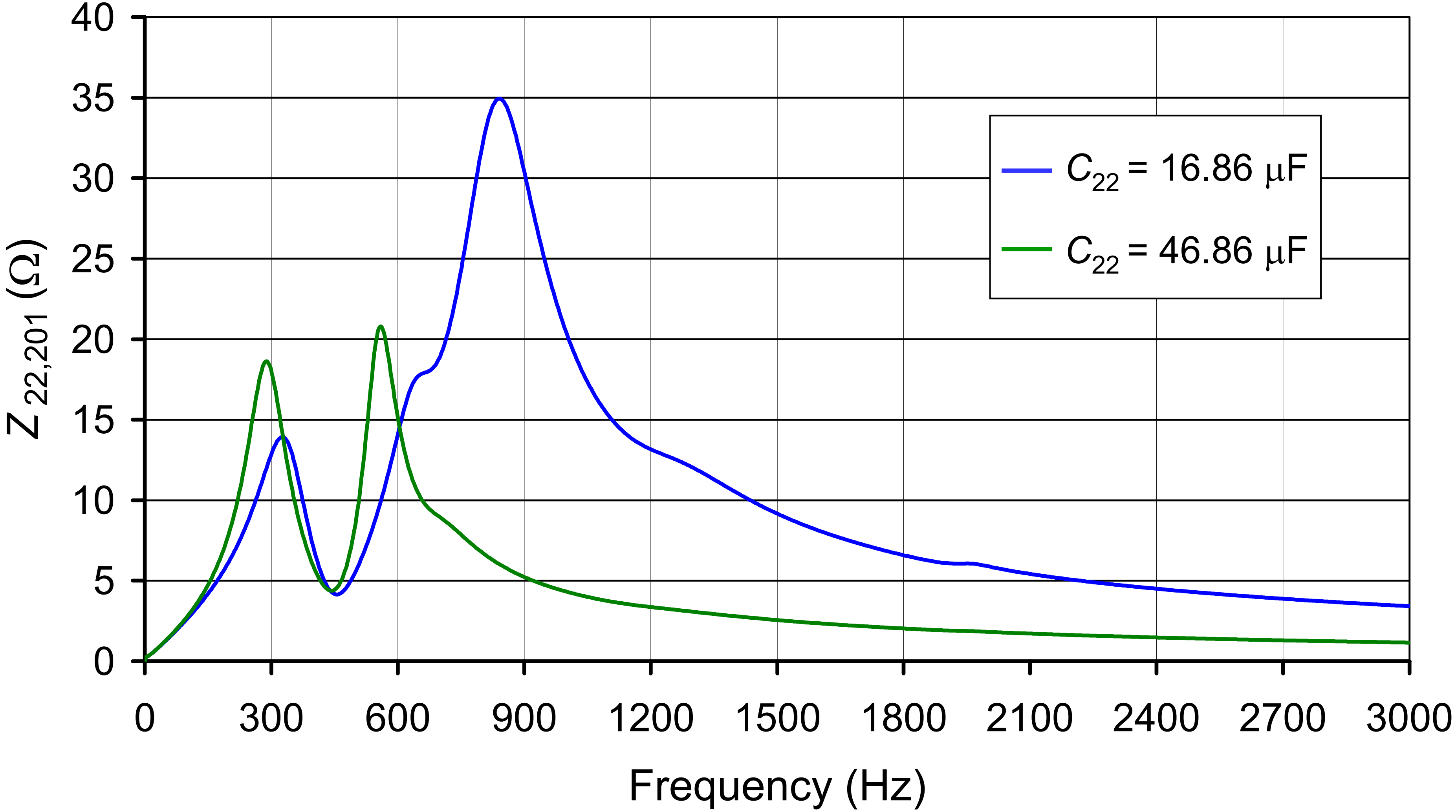
Frequency Associated with the Zeros of $Z_{201,22}$ and Their Sensitivities (Hz/μF) to Capacitor Changes for C_{22}=26.86 μF

	Z_1	Z_2	Z_3	Z_4	Z_5
f (Hz)→	450	670	1199	1914	1958
C_{11}	-3.018	-0.269	-0.900	-0.700	-98.199
C_{13}	-6.258	-3.683	-0.798	-0.257	-3.908
C_{22}	0.000	0.000	0.000	0.000	0.000
C_{31}	-1.063	-0.735	-47.545	-1.959	-2.180
C_{42}	-1.892	-7.970	-0.552	-24.396	-4.303
C_{43}	-2.227	-13.092	-4.552	-63.444	5.446

Option 2: Pole Shift Effects



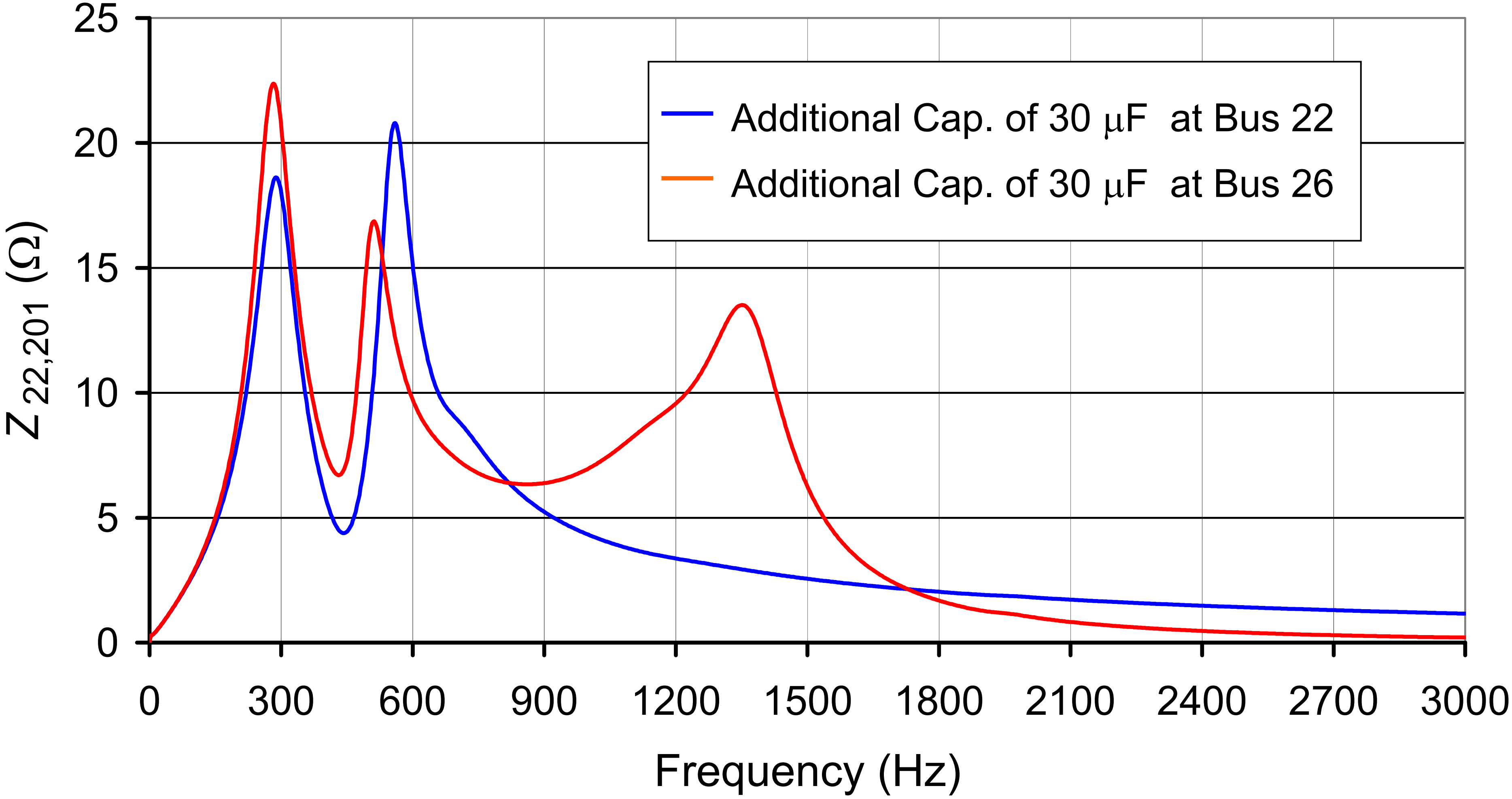
OPTION 2: FINAL RESULT



OPTION 2: PROPOSED SOLUTION

Harmonic Order	Original System (%)	Proposed Solution (%)	Limits (%)
11	5.20	2.89	3.5
13	7.48	1.85	3.0
23	1.40	0.368	1.5
25	1.18	0.330	1.5

REMARKS



25
20
15
10
5
0
0
300
600
900
1200
1500
1800
2100
2400
2700
3000
$Z_{22,201}$ (Ω)
Frequency (Hz)
Additional Cap. of 30 μF at Bus 22
Additional Cap. of 30 μF at Bus 26

CONCLUSIONS

- **Y**(s) is a natural evolution of the conventional **Y**($j\omega$) approach.
- **Y**(s) allows obtaining all the results produced by the **Y**($j\omega$) approach. Additionally, modal analysis can be performed using **Y**(s).
- Descriptor System approach can be used as a powerful complement of **Y**(s), since it allows the computation of the complete set of system poles and transfer function zeros at once.
- This paper is a tutorial example of using modal analysis to improve harmonic voltage distortions in electrical systems.

CONCLUSIONS

- Basically there are three forms of improving the harmonic performance of a system: Filtering harmonic currents, improving the performance of nonlinear loads and system modifications. This paper is a contribution for the third form.
- System modifications seems to be particularly suitable for reducing harmonic distortions in larger systems which have several and spread nonlinear loads.