IPST'99 – International Conference on Power Systems Transients, June 20-24, 1999, Budapest, Hungary

STUDYING HARMONIC PROBLEMS USING A DESCRIPTOR SYSTEM APPROACH

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Abstract – The harmonic voltage performance of a system depends on the location of its poles and zeros mainly with respect to the critical harmonic frequencies. Therefore the knowledge of the poles, zeros and their respective sensitivities to system parameters enables the identification of changes in the system which will reduce harmonic voltage levels. The method presented in [1], [2], based on state space formulation, allows this type of knowledge. Unfortunately, the construction of the state matrix for practical systems is not a simple task. Furthermore, the method in [1], [2] presents some limitations regarding network topology. This paper presents a method based on the descriptor system approach [3] which overcomes the computational difficulties associated with the state matrix method. The method properly deals with state variable redundancies and can be efficiently applied to large-scale networks of any topology.

Keywords: Descriptor System, Harmonics, Eigenvalues, Eigenvalue Sensitivities, Eigenvectors.

1. Introduction

Current programs for harmonic analysis in electrical systems require the inversion of the network admittance matrix, computed for numerous discrete values of frequency. This analysis is based on linear models and makes use of the superposition theorem, when considering multiple current harmonic sources [4], [5].

The main advantage of this method is that the admittance matrix is quite straightforward to build. This approach, however, does not allow the computation of the sensitivity of the resonance frequencies to the network parameters. Sensitivity analysis can play an important role in the elimination of undesirable resonances in the electrical system and as such it can be used as a powerful tool in harmonic studies [1], [2].

The method described in references [1] and [2], based on state variables, provides sensitivity analysis but the construction of the state matrix for practical systems is not a simple task. Furthermore the method assumes that two nodes can only be interconnected by an inductive element and that the capacitances are only connected between a node and the ground. Another limitation is that a capacitor bank must be modeled in every system bus.

The computational difficulties regarding the construction of the state matrix can be overcome with the use of the method proposed in [3], based on descriptor systems. The method properly deals with state variable redundancies and can be efficiently applied to large-scale networks of any topology.

The theoretical basis, comprising system modeling, harmonic impedance curves and sensitivity equations, will be presented in the next sections. Finally, results on a distribution system model will be described in order to validate and show the efficiency of the method.

2. Network Modeling

The behavior of any electrical network obeys three basic laws: Kirchoff law for currents, Kirchoff law for voltages and the inherent characteristics of each network element [6].

The Kirchoff laws contain the information on system topology and are represented by algebraic equations involving system variables (voltages and currents). Each algebraic equation determines a linear dependence among variables.

The system dynamics depends on the characteristic of its elements. In general, first order differential equations, in terms of currents and voltages, are used to represent inductive and capacitive elements. In this way, the inductive currents and capacitive voltages represent an obvious choice of state variables.

However, the construction of a dynamic model for the electrical network, based on state equations, is not so simple. By definition, the states form a minimum set of variables able to represent the dynamic behavior of a system [7]. In this way, a minimum set of inductive currents and capacitive voltages, which are linearly independent, must be determined.

This difficulty can be overcome by using the descriptor system (or partially dynamic system) to model the electrical network [3]. The network modeling by descriptor system uses all the inductive currents and all capacitive voltages as state variables. The algebraic constraints given by Kirchoff law for currents are also included in the model.

3. Single-phase RLC Series Branch

In general, the network modeling for studies regarding harmonic problems is done considering only the positive sequence network. For that reason, only single-phase models are necessary. Three-phase modeling for harmonic studies has been subject of research [8], but is not yet a common methodology for real system studies.

The single-phase RLC branch presented in Fig. 3.1, with frequency independent parameters, will be the basic element for network modeling in this work.

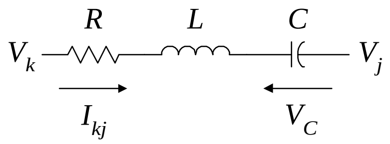

Fig. 3.1: RLC Series branch

The electrical behavior of this element can be described by a set of two ordinary differential equations of first order, as follows:

$$V_k - V_j = R \cdot I_{kj} + L \cdot \frac{dI_{kj}}{dt} + V_C \tag{3.1}$$

$$C \cdot \frac{dV_C}{dt} = I_{kj} \tag{3.2}$$

Equation (3.1) is general and holds for the particular cases where *L* or *R* are zero. However, when the parameter *C* does not exist in the branch, (3.2) must be replaced by:

$$V_C = 0 \tag{3.3}$$

4. Descriptor System for Single-phase RLC Networks

In this work, a single-phase RLC network will be represented by the interconnection of several elements like that one shown in Fig. 3.1. For each element, (3.1) and (3.2) can be written in matrix form as:

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_{kj} \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} -R & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{kj} \\ V_c \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot V_k + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot V_j \tag{4.1}$$

where the current I_{kj} through the inductor and the voltage V_c upon the capacitor were adopted as state variables. V_k and V_j are the voltages at nodes *k* and *j*, respectively. The dot over the variable represents its time derivative.

If there is no capacitance, (4.1) must be replaced by:

$$\begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_{kj} \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} -R & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{kj} \\ V_c \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot V_k + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot V_j \tag{4.2}$$

Note that transforming (4.1) into (4.2) is simply done by setting the value of *C* to zero and exchanging the elements in the last row of the state matrix.

It must be pointed out that I_{kj} in (4.1) is a current from node *k* to node *j*, i.e., a positive current injection at *j* and negative at *k*. The interconnection among the several elements will be given by the equations regarding the Kirchoff law for the current applied to each system node: the algebraic summation of the injected currents at a node must be zero. Therefore, the current I_{kj} must appear with a positive signal in the equation for the currents regarding the *j* node and with a negative signal in the equation regarding the *k* node. If the branch is connected to the ground $(j = 0 \text{ or } k = 0)$, the current through this branch will only be present in one equation.

Therefore, the modeling of the electric network will yield two differential equations for each system RLC branch and an algebraic equation (regarding the Kirchoff law for the currents) for each system node.

After modeling all RLC branches, the resultant descriptor system will have the following matrix structure:

$$\left[\begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array}\right] \cdot \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \hline \dot{\mathbf{v}}_{nodal} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array}\right] \cdot \begin{bmatrix} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \hline \mathbf{I} \end{bmatrix} \cdot \mathbf{i}_{nodal} \tag{4.3}$$

$$\mathbf{v}_{nodal} = \left[\mathbf{0}^T \mid \mathbf{I}\right] \cdot \begin{bmatrix} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{bmatrix} \tag{4.4}$$

Considering n_l as the number of RLC branches and n_n as the number of nodes of the electric network, then $\mathbf{A}_1$ and $\mathbf{T}_1$ have dimension $2 \cdot n_l \times 2 \cdot n_l$, $\mathbf{A}_2$ and $\mathbf{0}$ have dimension $2 \cdot n_l \times n_n$, $\mathbf{A}_3$ has dimension $n_n \times 2 \cdot n_l$ and $\mathbf{I}$ and $\mathbf{0}_q$ have dimension $n_n \times n_n$. The vector $\mathbf{x}_1$ has dimension $2 \cdot n_l$ and $\mathbf{v}_{nodal}$ and $\mathbf{i}_{nodal}$ have dimension n_n.

$\mathbf{T}_1$ is a diagonal matrix and $\mathbf{A}_1$ is block-diagonal, $\mathbf{A}_2$ and $\mathbf{A}_3$ are "incidence matrices" for the descriptor system. Symbols $\mathbf{0}$ and $\mathbf{0}_q$ denote zero matrices and $\mathbf{I}$ is the identity matrix.

The matrix equations (4.3) and (4.4) can be written in a more compact form as:

$$\mathbf{T} \cdot \dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} \tag{4.5}$$

$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x} \tag{4.6}$$

where:

$$\mathbf{T} = \left[\begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array}\right] \tag{4.7}$$

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array}\right] \tag{4.8}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \hline \mathbf{I} \end{bmatrix} \tag{4.9}$$

$$\mathbf{C} = \left[\mathbf{0}^T \mid \mathbf{I}\right] \tag{4.10}$$

$$\mathbf{u} = \mathbf{i}_{nodal} \tag{4.11}$$

$$\mathbf{y} = \mathbf{v}_{nodal} \tag{4.12}$$

Matrices $\mathbf{A}$ and $\mathbf{T}$ have dimension $(2 \cdot n_l + n_n) \times (2 \cdot n_l + n_n)$, $\mathbf{B}$ has dimension $(2 \cdot n_l + n_n) \times n_n$ and $\mathbf{C}$ has dimension $n_n \times (2 \cdot n_l + n_n)$. Vector $\mathbf{x}$ has dimension $(2 \cdot n_l + n_n)$ and $\mathbf{y}$ and $\mathbf{u}$ have dimension n_n.

5. Harmonic Impedance seen from a System Node

Applying the Laplace Transform to (4.5), one has:

$$\mathbf{X} = (s \cdot \mathbf{T} - \mathbf{A})^{-1} \cdot \mathbf{B} \cdot \mathbf{U} \tag{5.1}$$

where $\mathbf{X}$ and $\mathbf{U}$ are the Laplace transforms of $\mathbf{x}$ and $\mathbf{u}$, respectively.

The system impedance matrix, **Z,** is defined by:

$$\mathbf{Y} = \mathbf{Z} \cdot \mathbf{U} \tag{5.2}$$

where **Y** is the Laplace transform of **y**.

Applying the Laplace transform to (4.6), one has:

$$\mathbf{Y} = \mathbf{C} \cdot \mathbf{X} \tag{5.3}$$

Substituting (5.1) in (5.3), one has:

$$\mathbf{Y} = \mathbf{C} \cdot (s \cdot \mathbf{T} - \mathbf{A})^{-1} \cdot \mathbf{B} \cdot \mathbf{U} \tag{5.4}$$

From (5.2) and (5.4), one has:

$$\mathbf{Z} = \mathbf{C} \cdot (s \cdot \mathbf{T} - \mathbf{A})^{-1} \cdot \mathbf{B} \tag{5.5}$$

The self impedance seen from node k is given by the Z_{kk} element of the **Z** matrix. However, in accordance with (5.5) and with the equations numbered from (4.7) to (4.10) (definition equations of **A**, **B**, **C** and **T**), one concludes that the Z_{kk} element is equal to the $(2 \cdot n_l + k)$ diagonal element of $(s \cdot \mathbf{T} - \mathbf{A})^{-1}$. Thus:

$$Z_{kk} = \operatorname{diag}\left[(s \cdot \mathbf{T} - \mathbf{A})^{-1}\right]_{(2 \cdot n_l + k)} \qquad (5.6)$$

On the other hand, the inverse of $(s \cdot \mathbf{T} - \mathbf{A})$ can be given by:

$$(s \cdot \mathbf{T} - \mathbf{A})^{-1} = \frac{\operatorname{adj}(s \cdot \mathbf{T} - \mathbf{A})}{\det(s \cdot \mathbf{T} - \mathbf{A})} \qquad (5.7)$$

Consider $\mathbf{T}_k$ and $\mathbf{A}_k$ as the matrices obtained by canceling the $2 \cdot n_l + k$ row and column of the matrices **T** and **A**, respectively. Thus, the $2 \cdot n_l + k$ diagonal element of $(s \cdot \mathbf{T} - \mathbf{A})^{-1}$ is given by:

$$Z_{kk} = \operatorname{diag}\left[(s \cdot \mathbf{T} - \mathbf{A})^{-1}\right]_{(2 \cdot n_l + k)} = \frac{\det(s \cdot \mathbf{T}_k - \mathbf{A}_k)}{\det(s \cdot \mathbf{T} - \mathbf{A})} \qquad (5.8)$$

Equation (5.8) is a generalization of its counterpart presented in [1] and [2]. It shows that:

- The system poles correspond to the generalized eigenvalue problem associated with the matrix pair $\{\mathbf{A}, \mathbf{T}\}$:

$$\det(s \cdot \mathbf{T} - \mathbf{A}) = 0 \Leftrightarrow \mathbf{A} \cdot \mathbf{v}_i = \lambda_i \cdot \mathbf{T} \cdot \mathbf{v}_i \qquad (5.9)$$

- The zeros, associated with the self impedance of node k, correspond to the generalized eigenvalue problem associated with the matrix pair $\{\mathbf{A}_k, \mathbf{T}_k\}$:

$$\det(s \cdot \mathbf{T}_k - \mathbf{A}_k) = 0 \Leftrightarrow \mathbf{A}_k \cdot \mathbf{v}_i = \lambda_i \cdot \mathbf{T}_k \cdot \mathbf{v}_i \qquad (5.10)$$

where λ_i are the generalized eigenvalues associated with the pair $\{\mathbf{A}, \mathbf{T}\}$ or $\{\mathbf{A}_k, \mathbf{T}_k\}$ and $\mathbf{v}_i$ are their associated generalized eigenvectors.

6. Example of Electric Network Modeling

A simple RLC circuit with state redundancy (see Fig. 6.1) will be used to describe the network modeling, by means of the descriptor system technique.

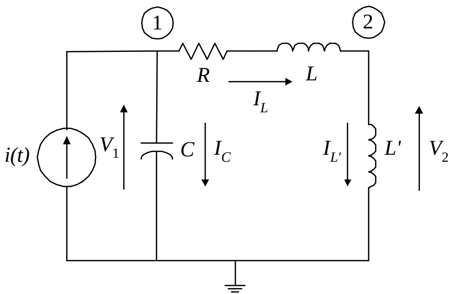

Fig. 6.1: RLC Circuit with state redundancy

The state redundancy of this RLC circuit was artificially introduced by splitting its inductance into two parts.

It must be pointed out that the RLC circuit shown in Fig. 6.1 is a particular case of the RLC circuit shown in Fig. 6.2. When C_{12} and C_2 are neglected and $R_1 = L_1 = R_2 = 0$, $C_1 = C$, $R_{12} = R$, $L_{12} = L$ and $L_2 = L'$ the circuit shown in Fig. 6.2 becomes equivalent to that shown in Fig. 6.1.

For an easier understanding, the modeling of the circuit shown in Fig. 6.1will be derived from the modeling of the circuit shown in Fig. 6.2.

In this modeling, the inductive currents and capacitive voltages are chosen as state variables. A matrix equation in the form of (4.1) is written for each branch. The branch connections are taken into account by the Kirchoff law for the currents applied to the circuit nodes.

For the circuit shown in Fig. 6.2 one has:

$$i - I_{L1} - I_{L12} = 0 \qquad (6.1)$$

$$I_{L12} - I_{L2} = 0 \qquad (6.2)$$

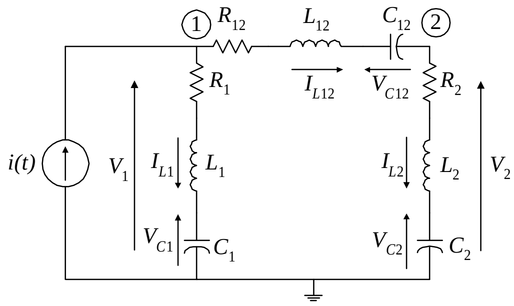

Fig. 6.2: Generalized RLC circuit

Joining the matrix equations relative to each circuit branch and (6.1) and (6.2), yield:

$$\begin{bmatrix} L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_{L1} \\ \dot{V}_{C1} \\ \dot{I}_{L2} \\ \dot{V}_{C2} \\ \dot{I}_{L12} \\ \dot{V}_{C12} \\ \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} =$$

$$\begin{bmatrix} -R_1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_2 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_{12} & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{L1} \\ V_{C1} \\ I_{L2} \\ V_{C2} \\ I_{L12} \\ V_{C12} \\ V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot i(t) \qquad (6.3)$$

The node voltage vector is given by:

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{L1} \\ V_{C1} \\ I_{L2} \\ V_{C2} \\ I_{L12} \\ V_{C12} \\ V_1 \\ V_2 \end{bmatrix} \qquad (6.4)$$

The model of the circuit shown in Fig. 6.1 is accomplished simply by setting $R_1 = L_1 = R_2 = 0$ and neglecting C_{12} and C_2 which, regarding the items 3 and 4, implies in modifying (6.3) to:

$$\left[\begin{array}{cc|cc|cc|cc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & L_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \cdot \left[\begin{array}{c} \dot{I}_{L1} \\ \dot{V}_{C1} \\ \hline \dot{I}_{L2} \\ \dot{V}_{C2} \\ \hline \dot{I}_{L12} \\ \dot{V}_{C12} \\ \hline \dot{V}_1 \\ \dot{V}_2 \end{array}\right] =$$

$$\left[\begin{array}{cc|cc|cc|cc} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -R_{12} & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \cdot \left[\begin{array}{c} I_{L1} \\ V_{C1} \\ \hline I_{L2} \\ V_{C2} \\ \hline I_{L12} \\ V_{C12} \\ \hline V_1 \\ V_2 \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 1 \\ 0 \end{array}\right] \cdot i(t) \qquad (6.5)$$

where

$$\begin{array}{ll} C_1 = C & I_{L1} = I_C \\ R_{12} = R & I_{L12} = I_L \\ L_{12} = L & I_{L2} = I_{L'} \\ L_2 = L' & V_{C1} = V_1 \\ & V_{C12} = V_{C2} = 0 \end{array} \qquad (6.6)$$

The dynamic model of the circuit shown in Fig. 6.1, is therefore, given by (6.4) and (6.5), considering the relations given in (6.6).

7. Eigenvalue Sensitivity

Considering $\lambda_1, \lambda_2, \ldots, \lambda_n$ as the generalized eigenvalues associated with the matrices $\{\mathbf{A}, \mathbf{T}\}$, one has:

$$\mathbf{A} \cdot \mathbf{v}_i = \lambda_i \cdot \mathbf{T} \cdot \mathbf{v}_i \qquad (7.1)$$

where $\mathbf{v}_i$ is the *right (column) eigenvector* of $\{\mathbf{A}, \mathbf{T}\}$ associated with the eigenvalue λ_i.

Similarly, the *left (row) eigenvector* $\mathbf{w}_i$ associated with the eigenvalue λ_i can be defined by:

$$\mathbf{w}_i \cdot \mathbf{A} = \lambda_i \cdot \mathbf{w}_i \cdot \mathbf{T} \qquad (7.2)$$

Differentiating (7.1) with respect to α_j (a system parameter) yields:

$$\frac{\partial \mathbf{A}}{\partial \alpha_j} \cdot \mathbf{v}_i + \mathbf{A} \cdot \frac{\partial \mathbf{v}_i}{\partial \alpha_j} = \frac{\partial \lambda_i}{\partial \alpha_j} \cdot \mathbf{T} \cdot \mathbf{v}_i + \lambda_i \cdot \frac{\partial \mathbf{T}}{\partial \alpha_j} \cdot \mathbf{v}_i + \lambda_i \cdot \mathbf{T} \cdot \frac{\partial \mathbf{v}_i}{\partial \alpha_j} \qquad (7.3)$$

Premultiplying (7.3) by $\mathbf{w}_i$ and using (7.2), one obtains

$$\frac{\partial \lambda_i}{\partial \alpha_j} = \frac{1}{C_i} \cdot \mathbf{w}_i \cdot \left(\frac{\partial \mathbf{A}}{\partial \alpha_j} - \lambda_i \cdot \frac{\partial \mathbf{T}}{\partial \alpha_j} \right) \cdot \mathbf{v}_i \qquad (7.4)$$

where:

$$C_i = \mathbf{w}_i \cdot \mathbf{T} \cdot \mathbf{v}_i \qquad (7.5)$$

Equation (7.4) is a generalization of the well-known sensitivity equation for the eigenvalue λ_i with respect to a system parameter α_j [1], [2].

In most cases, the normalization of the sensitivities is useful. This is obtained by multiplying the eigenvalue sensitivity $\frac{\partial \lambda_i}{\partial \alpha_j}$ by the system parameter α_j [1], [2].

Let α_j^0 be the initial value of the system parameter α_j. Thus the variation of the eigenvalue λ_i as a function of the parameter variation $\Delta\alpha_j$ and of the normalized sensitivity is given in a first approximation by:

$$\Delta\lambda_i = \alpha_j^0 \cdot \frac{\partial \lambda_i}{\partial \alpha_j}\left(\alpha_j^0\right) \cdot \frac{\Delta\alpha_j}{\alpha_j^0} \qquad (7.6)$$

The symbol $\frac{\partial \lambda_i}{\partial \alpha_j}\left(\alpha_j^0\right)$ denotes that the eigenvalue sensitivity has been computed for a parameter value of α_j^0.

Equation (7.6) can be easily generalized for simultaneous variation of several system parameters.

8. Test System

The test system utilized has been taken from [2], and is shown in Fig. 8.1.

This system can be modeled by the interconnection of several series RLC branches, as shown in Fig. 8.2.

The system frequency is 50 Hz and the values of its elements are given in Table 8.1. L_{12} and R_{12} represent the inductance and resistance of the equivalent series association of the line *LT 1-2* with the transformer *T2*. Similarly, L_{13} and R_{13} represent the inductance and resistance of the equivalent series association of the line *LT 1-3* with the transformer *T3*.

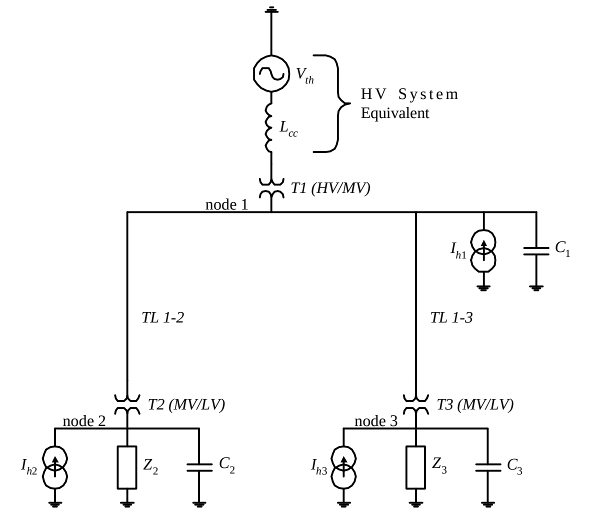

Fig. 8.1: Test system

V_{th}	: Thévenin voltage.
L_{cc}	: Short-circuit inductance of the HV system.
T1	: HV/MV Transformer.
T2, T3	: MV/LV Transformers.
TL 1-2	: Transmission line connecting node 1 to transformer *T2*.
TL 1-3	: Transmission line connecting node 2 to transformer *T3*.
C_1, C_2, C_3	: Capacitor banks connected to nodes 1, 2 and 3, respectively.
Z_2, Z_3	: Load impedances connected to nodes 2 and 3, respectively.
I_{h1}, I_{h2}, I_{h3}	: Harmonic current sources connected to nodes 1, 2 and 3, respectively.

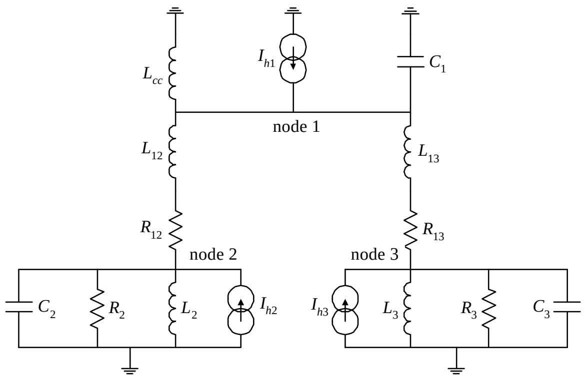

Fig. 8.2: System modeling

Table 8.1: System parameter values

Inductance (mH)		Resistance (Ω)		Capacitance (μF)	
L_{cc}	8.0	R_2	80.0	C_1	23.9
L_2	424.0	R_3	133.0	C_2	8.0
L_3	531.0	R_{12}	0.46	C_3	11.9
L_{12}	9.7	R_{13}	0.55		
L_{13}	11.9				

The matrices **A** and **T** have dimension $2 \cdot n_l + n_n$, where n_l represents the number of RLC branches and n_n the number of nodes of the electric network. For the test system, the order these matrices is 23. The sparse structure of the generalized state matrix (**A**), for the test system, is depicted in Fig. 8.3.

The constant nz represents the number of nonzero elements, and is shown at the bottom of Fig. 8.3.

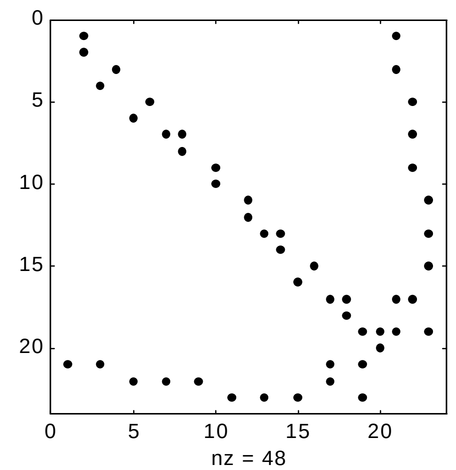

Fig. 8.3: Sparse structure of matrix **A** for Test System

8.1 Calculation of Poles, Zeros and Sensitivities

Although the order of the matrices **A** and **T** is 23, the test system is actually of eighth-order. Therefore, 15 generalized eigenvalues of infinite modulus corresponding to the algebraic equations are obtained.

For matrices $\mathbf{A}_k$ and $\mathbf{T}_k$ there are also 15 generalized eigenvalues of infinite modulus but only 7 of finite modulus. The values of the finite generalized eigenvalues associated to $\{\mathbf{A}, \mathbf{T}\}$ (poles) and to $\{\mathbf{A}_k, \mathbf{T}_k, k = 1,2,3\}$ (zeros), are shown in Table 8.2.

The frequencies of poles (parallel resonance) and zeros (series resonance), given by the imaginary part of the complex conjugate eigenvalues, and their sensitivities with respect to the inductances and capacitances of the studied system are presented in Table 8.3. Note there is a 2π factor between the frequency values shown in Table 8.2 and Table 8.3, since the latter is given in Hz.

Table 8.2: Generalized eigenvalues

$\{\mathbf{A}, \mathbf{T}\}$	$\{\mathbf{A}_1, \mathbf{T}_1\}$	$\{\mathbf{A}_2, \mathbf{T}_2\}$	$\{\mathbf{A}_3, \mathbf{T}_3\}$
-345.9 $\pm j$4535.6	-338.5 $\pm j$2670.9	-93.7 $\pm j$3975.6	-398.4 $\pm j$4424.9
-507.0 $\pm j$3069.1	-804.4 $\pm j$3550.6	-255.5 $\pm j$2084.9	-415.3 $\pm j$2402.1
-290.1 $\pm j$1583.6	-1.0	-26.2	-27.8
-1.0	-1.1	-1.0	-1.0
-1.0	0.0	0.0	0.0

Table 8.3: Resonance frequency sensitivities

	Poles			Zeros					
				Node 1		Node 2		Node 3	
	1	2	3	1	2	1	2	1	2
f(Hz)	252	488	722	425	565	332	633	382	704
L_{cc}	633	68	312	0	0	302	501	550	292
L_2	18	22	11	0	41	0	0	33	15
L_3	22	12	1	30	0	29	5	0	0
L_{12}	11	493	1551	0	1820	248	415	232	1819
L_{13}	119	949	492	1324	0	470	1080	368	199
C_1	284	74	1295	0	0	237	1523	533	1188
C_2	158	732	697	0	1689	0	0	685	912
C_3	339	719	178	1317	0	**799**	452	0	0

It must be pointed out that all the results presented here are in good agreement with those presented in [2].

8.2 Shifting Poles and Zeros

As shown in Table 8.3 the pole 1 is located at 252 Hz. It can cause problems at any system bus since an injection of fifth-harmonic current can generate high levels of harmonic distortions.

A possible solution consists in bringing the zero 1 seen from node 2 closer to the frequency of 250 Hz.

The highlighted sensitivity results in Table 8.3 indicate that changes in parameter C_3 will cause the largest shifts in the chosen zero.

Using (7.6) a value of $C_3 = 19.56$ μF was obtained.

The impedance magnitudes seen from bus 2 associated with the original and the new value of C_3 are shown in Fig. 8.4.

The frequencies of the poles and zeros, for the new value of C_3, are presented in Table 8.4. As shown in this table, the frequency of the pole 1 seen from node 2 is 270 Hz instead of 250 Hz. A more accurate technique for shifting poles and zeros based on Newton-Raphson method is currently being developed.

For this new value of C_3 there was a reduction of 47% in the impedance magnitude at 250 Hz.

Table 8.4: New frequencies of Poles and Zeros

	Poles			Zeros					
				Node 1		Node 2		Node 3	
	1	2	3	1	2	1	2	1	2
f(Hz)	221	442	713	332	565	270	606	382	704

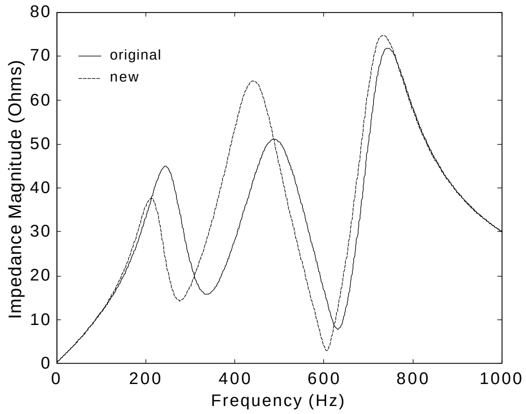

Fig. 8.4: Impedance modulus seen from node 2

9. Conclusion

The state space method [1], [2] and the proposed descriptor system method allow obtaining all the results produced by the traditional method which is based on nodal admittance matrices computed at various discrete values of frequency within the range of interest. Furthermore, these methods allow:

- Identification of elements mostly involved in specific resonances.
- Determination of the necessary changes in system elements in order to shift the location of poles and/or zeros to desired positions.
- Optimum allocation of capacitor banks and/or passive filters.

The state space method in [1], [2] has, however, some limitations regarding its ability to model practical networks. The proposed descriptor system method overcomes these limitations and offers the following advantages:

- Simple and efficient computational implementation.
- Ability to model systems of any topology and containing state variable redundancies.
- Applicability to large-scale networks, due to the very sparse matrices involved and the availability of powerful sparse eigensolution algorithms applied to descriptor systems [3], [9], [10].

10. Bibliography

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