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MODAL ANALYSIS of ELETROMAGNETIC TRANSIENTS IN AC NETWORKS HAVING LONG TRANSMISSION LINES

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Introduction

- Linear dynamic systems may be studied using modal analysis, that is based on the computation of the system poles.
- Modal analysis is very useful to obtain structural information on the system and to propose means to improve system behavior.
- Modal analysis of electromagnetic transients in system having long transmission lines is efficiently done using s-domain models with algorithms developed in previous authors' work.
- Modal analysis applied to electromagnetic transients may be used to obtain modal equivalents, determine elements responsible for overvoltages, determine the most effective parameter changes to improve dynamic performance, etc.
- The paper will present the basis for the use of modal analysis in electromagnetic transients and results on a synthetic power system where a modal equivalent will be used to efficiently determine the maximum system overvoltage.

Formulations for modal analysis

State-space:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

$$s\,\mathbf{x}(s) = \mathbf{A}\,\mathbf{x}(s) + \mathbf{B}\,\mathbf{u}(s)$$
$$\mathbf{y}(s) = \mathbf{C}\,\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)$$

Descriptor system:

$$\mathbf{T}\,\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

$$s\,\mathbf{T}\,\mathbf{x}(s) = \mathbf{A}\,\mathbf{x}(s) + \mathbf{B}\,\mathbf{u}(s)$$
$$\mathbf{y}(s) = \mathbf{C}\,\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)$$

s-Domain:

Not represented

$$\mathbf{Y}(s)\,\mathbf{x}(s) = \mathbf{B}(s)\,\mathbf{u}(s)$$
$$\mathbf{y}(s) = \mathbf{C}(s)\,\mathbf{x}(s) + \mathbf{D}(s)\,\mathbf{u}(s)$$

Linear analysis using s-domain formulation

s-Domain formulation:

$$\mathbf{Y}(s)\cdot\mathbf{x}(s)=\mathbf{b}\cdot u(s)$$

$$y(s)=\mathbf{c}^t\cdot\mathbf{x}(s)$$

Transfer function:

$$G(s)=\frac{y(s)}{u(s)}=\mathbf{c}^t\cdot\left[\mathbf{Y}(s)\right]^{-1}\cdot\mathbf{b}$$

Frequency response: $G(j\,\omega)$

Modal analysis:

Pole: $\det\left[\mathbf{Y}(\lambda_i)\right]=0$ $\qquad G(\lambda_i)\rightarrow\infty$

Zero: $G(z_i)=0$

Tools: root-locus, sensitivities, modal time response, etc.

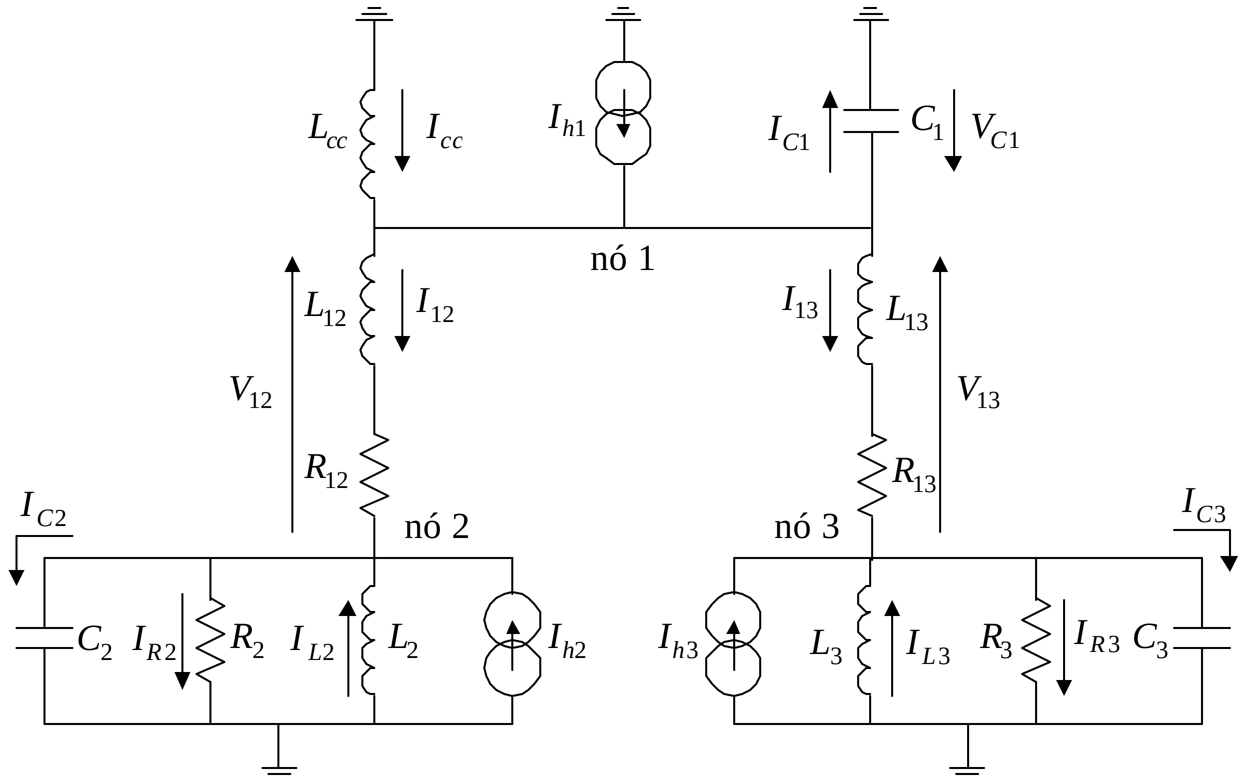
Nodal Admittance method (s-domain formulation)

$$\mathbf{Y}(s)=\begin{bmatrix} \dfrac{1}{s\,L_{cc}}+\dfrac{1}{R_{12}+s\,L_{12}}+s\,C_1+\dfrac{1}{R_{13}+s\,L_{13}} & -\dfrac{1}{R_{12}+s\,L_{12}} & -\dfrac{1}{R_{13}+s\,L_{13}} \\ -\dfrac{1}{R_{12}+s\,L_{12}} & \dfrac{1}{R_2}+\dfrac{1}{s\,L_2}+s\,C_2+\dfrac{1}{R_{12}+s\,L_{12}} & 0 \\ -\dfrac{1}{R_{13}+s\,L_{13}} & 0 & \dfrac{1}{R_3}+\dfrac{1}{s\,L_3}+s\,C_3+\dfrac{1}{R_{13}+s\,L_{13}} \end{bmatrix}$$

Network model:

$$\mathbf{Y}(s)\cdot\mathbf{V}=\mathbf{b}\cdot I_i$$

$$V_i=\mathbf{c}^t\cdot\mathbf{V}$$



Y(s) Derivative

Newton-type algorithms, previously proposed by the authors, use the s-derivative of Y(s) to efficiently compute system poles.

$$\mathbf{Y}(s)=\begin{bmatrix} \frac{1}{s\,L_{cc}}+\frac{1}{R_{12}+s\,L_{12}}+s\,C_1+\frac{1}{R_{13}+s\,L_{13}} & -\frac{1}{R_{12}+s\,L_{12}} & -\frac{1}{R_{13}+s\,L_{13}} \\ -\frac{1}{R_{12}+s\,L_{12}} & \frac{1}{R_2}+\frac{1}{s\,L_2}+s\,C_2+\frac{1}{R_{12}+s\,L_{12}} & 0 \\ -\frac{1}{R_{13}+s\,L_{13}} & 0 & \frac{1}{R_3}+\frac{1}{s\,L_3}+s\,C_3+\frac{1}{R_{13}+s\,L_{13}} \end{bmatrix}$$

$$\frac{d\mathbf{Y}}{ds}=\begin{bmatrix} -\frac{1}{s^2\,L_{cc}}-\frac{L_{12}}{(R_{12}+s\,L_{12})^2}+C_1-\frac{L_{13}}{(R_{13}+s\,L_{13})^2} & \frac{L_{12}}{(R_{12}+s\,L_{12})^2} & \frac{L_{13}}{(R_{13}+s\,L_{13})^2} \\ \frac{L_{12}}{(R_{12}+s\,L_{12})^2} & -\frac{1}{s^2\,L_2}+C_2-\frac{L_{12}}{(R_{12}+s\,L_{12})^2} & 0 \\ \frac{L_{13}}{(R_{13}+s\,L_{13})^2} & 0 & -\frac{1}{s^2\,L_3}+C_3-\frac{L_{13}}{(R_{13}+s\,L_{13})^2} \end{bmatrix}$$

S-domain model of transmission lines

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_c \coth(\gamma \cdot l) & -y_c \operatorname{csch}(\gamma \cdot l) \\ -y_c \operatorname{csch}(\gamma \cdot l) & y_c \coth(\gamma \cdot l) \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_c = \sqrt{\frac{Y_u(s)}{Z_u(s)}}$$

$$\gamma = \sqrt{Z_u(s) \cdot Y_u(s)}$$

Basic Concepts on Modal Analysis

Transfer Function:

$$Y(s) = G(s)\,U(s)$$

Partial Fraction Form:

$$G(s) = \sum_i \frac{R_i}{s - \lambda_i} + d$$

Impulse time response (*U*(*s*) = 1):

$$y(t) = \sum_i R_i\, e^{\lambda_i t} + d\;\delta(t)$$

Dominant Pole Algorithm

$$\begin{bmatrix} \mathbf{Y}(\lambda^{(k)}) & -\mathbf{b} \\ \mathbf{c}^t & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}^{(k)} \\ u_1^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{Y}(\lambda^{(k)})^{\,t} & \mathbf{c} \\ -\mathbf{b}^t & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w}^{(k)} \\ u_2^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

Transfer Function Residue: $R_i^{(k+1)} = -\dfrac{1}{\left[\mathbf{w}^{(k)}\right]^t \cdot \dfrac{d\mathbf{Y}(\lambda^{(k)})}{ds} \cdot \mathbf{v}^{(k)}}$

Pole Correction: $\Delta\lambda^{(k)} = -u_1^{(k)} \cdot R_1^{(k+1)}$

New estimate for pole: $\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$

Basic Concepts on Modal Analysis

Generic input:

$$Y(s) = G(s) \cdot U(s) = \left(\sum_i \frac{R_i}{s - \lambda_i} + d \right) \cdot \left(\sum_i \frac{R_i^u}{s - \lambda_i^u} + d^u \right)$$

Partial Fraction Form:

$$Y(s) = \sum_i \frac{\overline{R}_i}{s - \lambda_i} + \sum_i \frac{\overline{R}_i^u}{s - \lambda_i^u} + \overline{d}$$

where:

$$\overline{R}_i = R_i \cdot G(\lambda_i)$$

$$\overline{R}_i^u = R_i^u \cdot U(\lambda_i^u)$$

$$\overline{d} = d \cdot d^u$$

Time Response:

$$y(t) = \sum_i \overline{R}_i \, e^{\lambda_i t} + \sum_i \overline{R}_i^u \, e^{\lambda_i^u t} + \overline{d} \, \delta(t)$$

Basic Concepts on Modal Analysis

Sine input:

$$u(t) = \sin(\omega t) \Rightarrow U(s) = \frac{\omega}{s^2 + \omega^2}$$

Cosine input:

$$u(t) = \cos(\omega t) \Rightarrow U(s) = \frac{s}{s^2 + \omega^2}$$

Sinusoidal input:

$$u(t) = A \cdot \sin(\omega t + \theta) = (A \cdot \cos\theta) \cdot \sin(\omega t) + (A \cdot \sin\theta) \cdot \cos(\omega t)$$

$$U(s) = \frac{(A \cdot \cos\theta) \cdot \omega + (A \cdot \sin\theta) \cdot s}{s^2 + \omega^2}$$

Modal Equivalent

Output variable for generic input:

$$Y(s) = \sum_i \frac{\overline{R}_i}{s-\lambda_i} + \sum_i \frac{\overline{R}_i^u}{s-\lambda_i^u} + \overline{d}$$

$$\lambda_i = \sigma_i + j \cdot \omega_i$$

$$y(t) = \sum_i \overline{R}_i \; e^{\lambda_i t} + \sum_i \overline{R}_i^u \; e^{\lambda_i^u t} + \overline{d} \; \delta(t)$$

s-domain component for a complex conjugate poles:

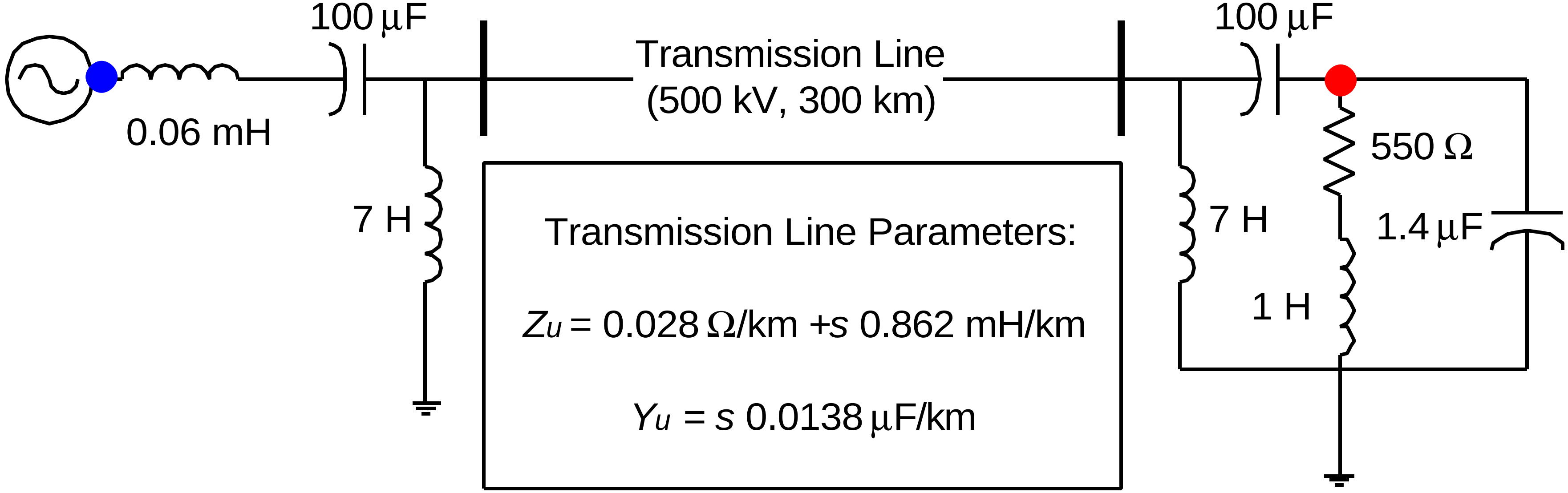
$$Y_i(s) = \frac{R_i}{s-\lambda_i} + \frac{R_i^*}{s-\lambda_i^*}$$

Modal component for a complex conjugate pair:

$$y_i(t) = R_i \; e^{\lambda_i t} + R_i^* \; e^{\lambda_i^* t} = 2 \cdot \left| R_i \right| \cdot e^{\sigma_i t} \cdot \cos(\omega_i \; t + \theta_R)$$

$$\theta_R = \arg(R_i)$$

Synthetic Test System

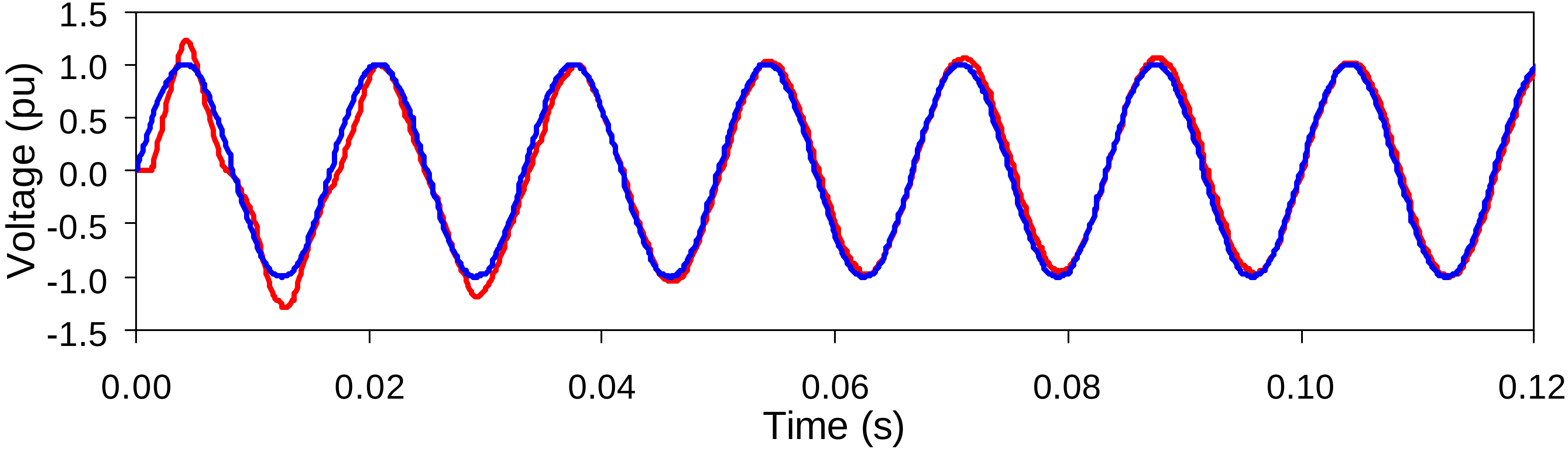


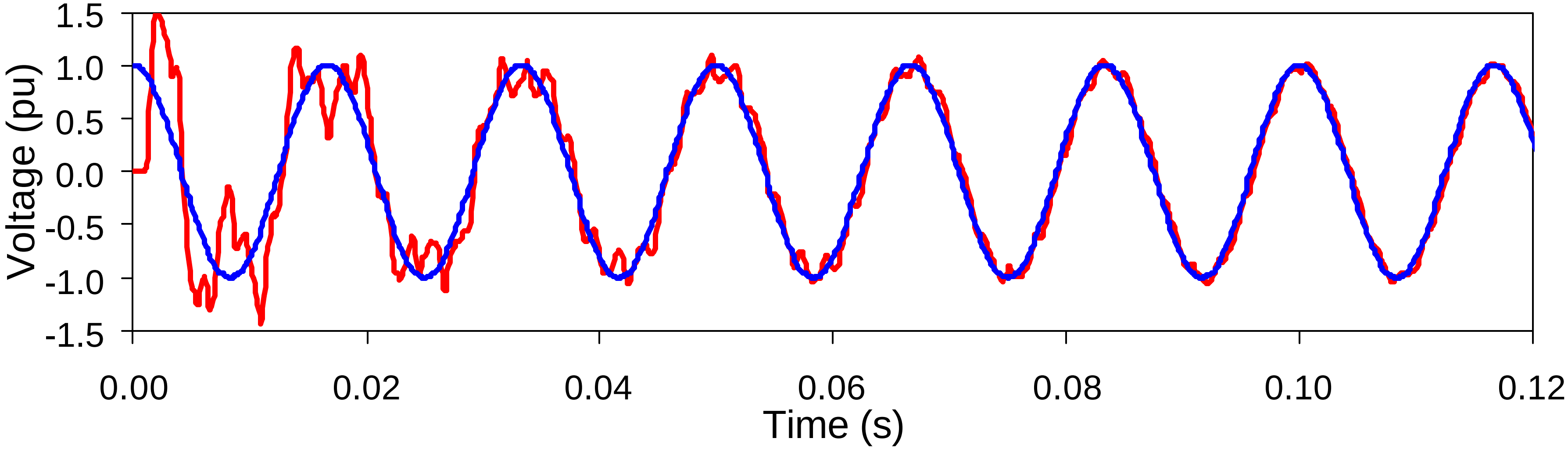
Transmission Line Parameters:

$Z_u = 0.028\ \Omega/\text{km} + s\ 0.862\ \text{mH/km}$

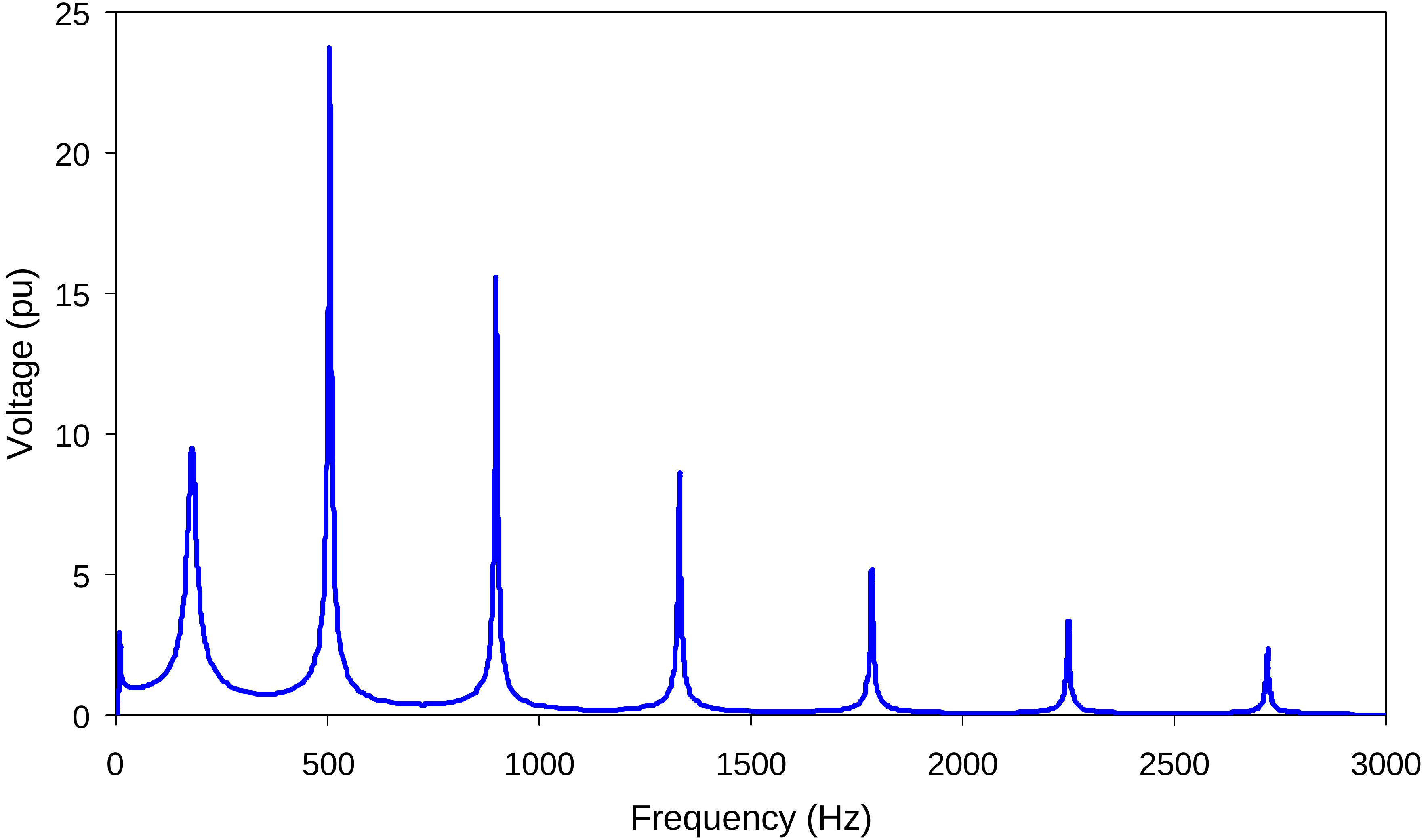
$Y_u = s\ 0.0138\ \mu\text{F/km}$

Time Simulation for sine and cosine input

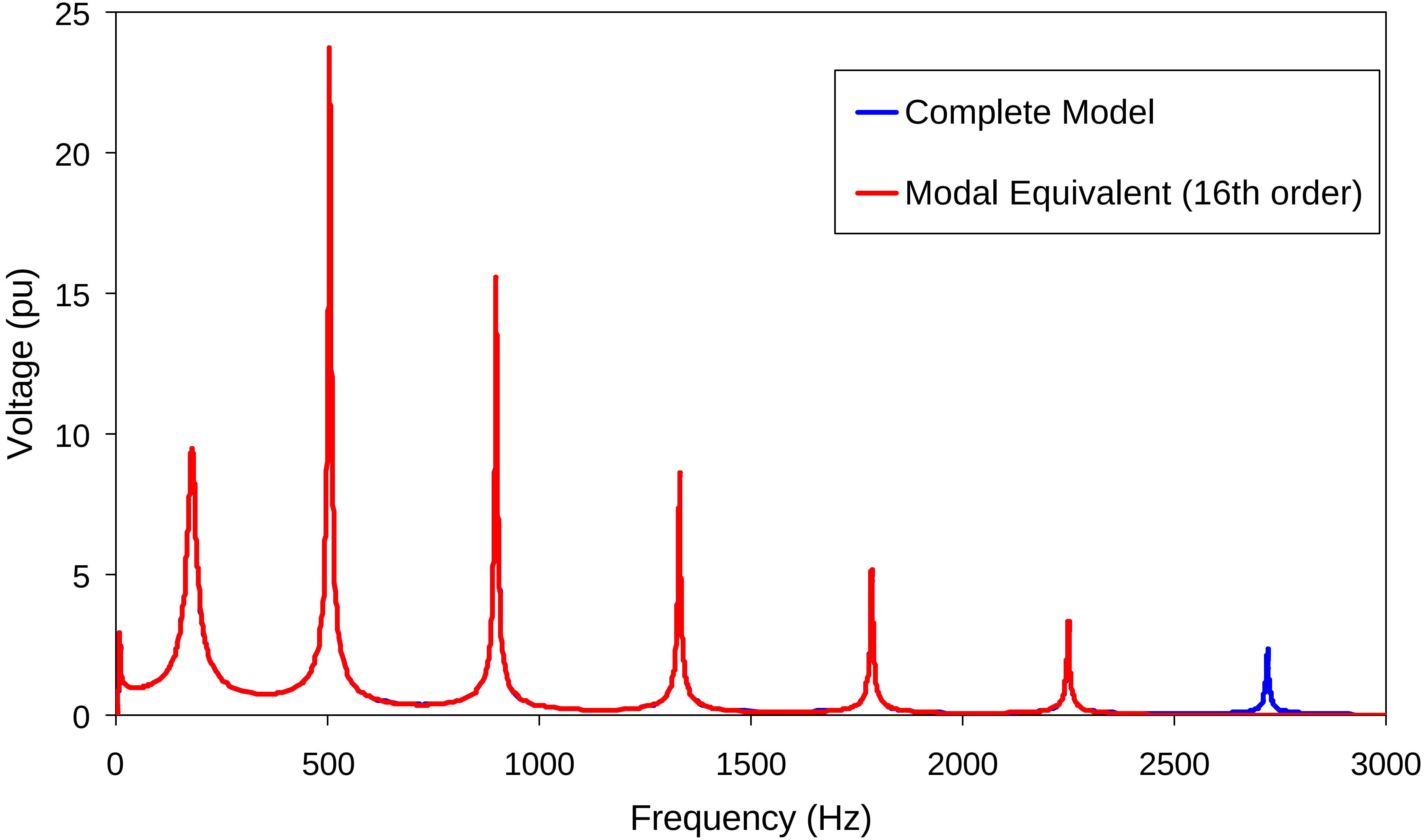




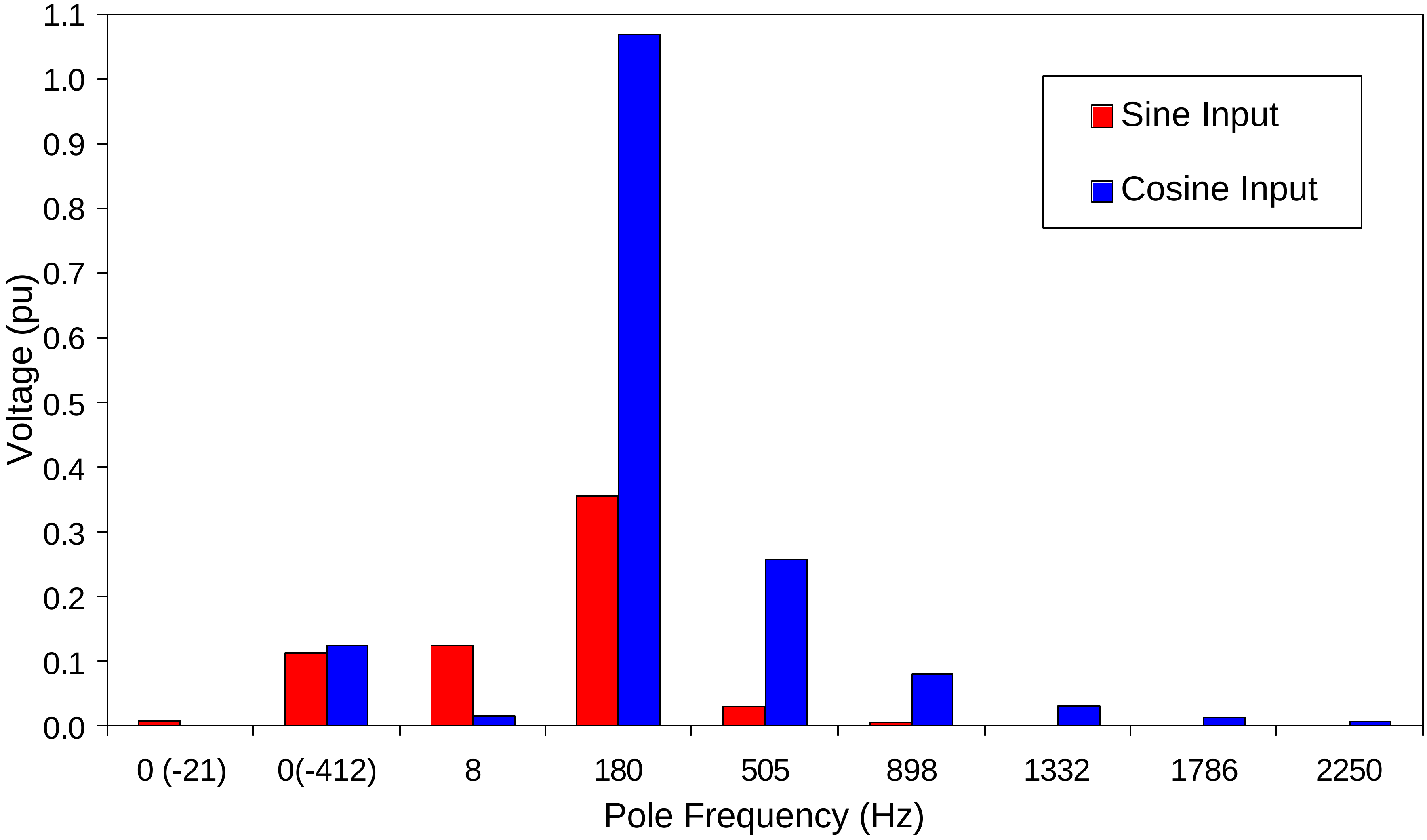
Frequency response



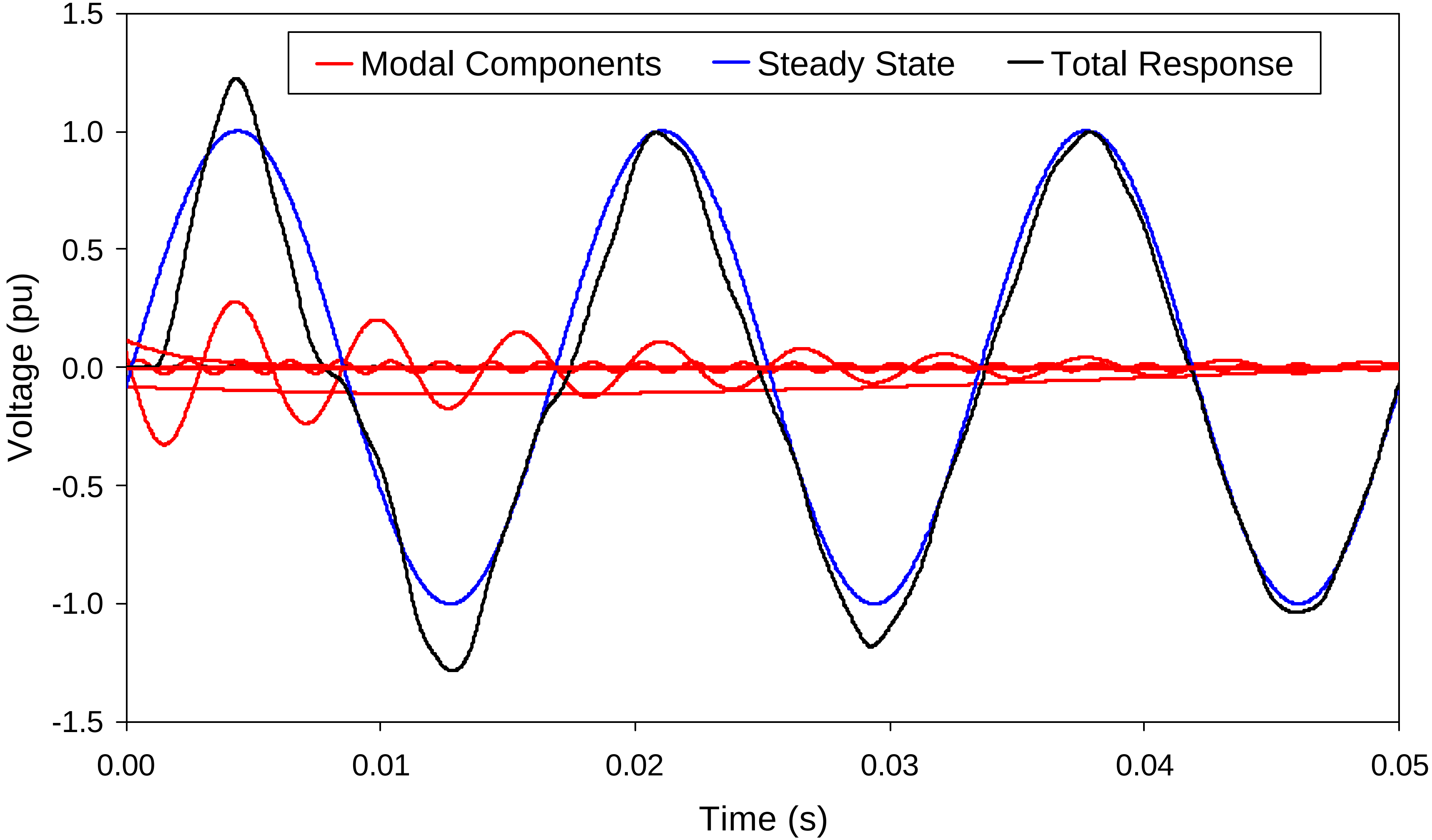
Modal Equivalent



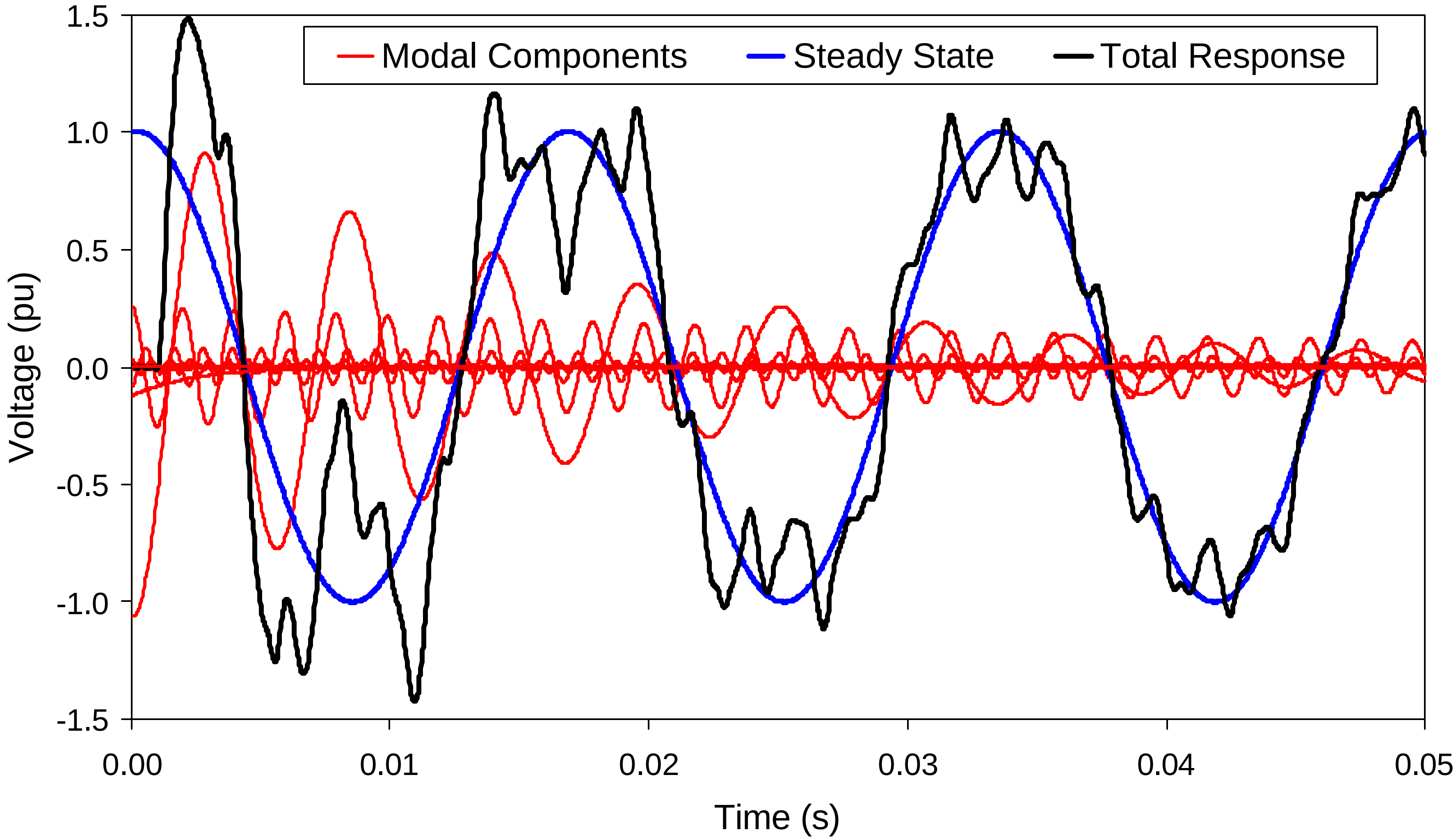
Modal Components for Sine and Cosine Input



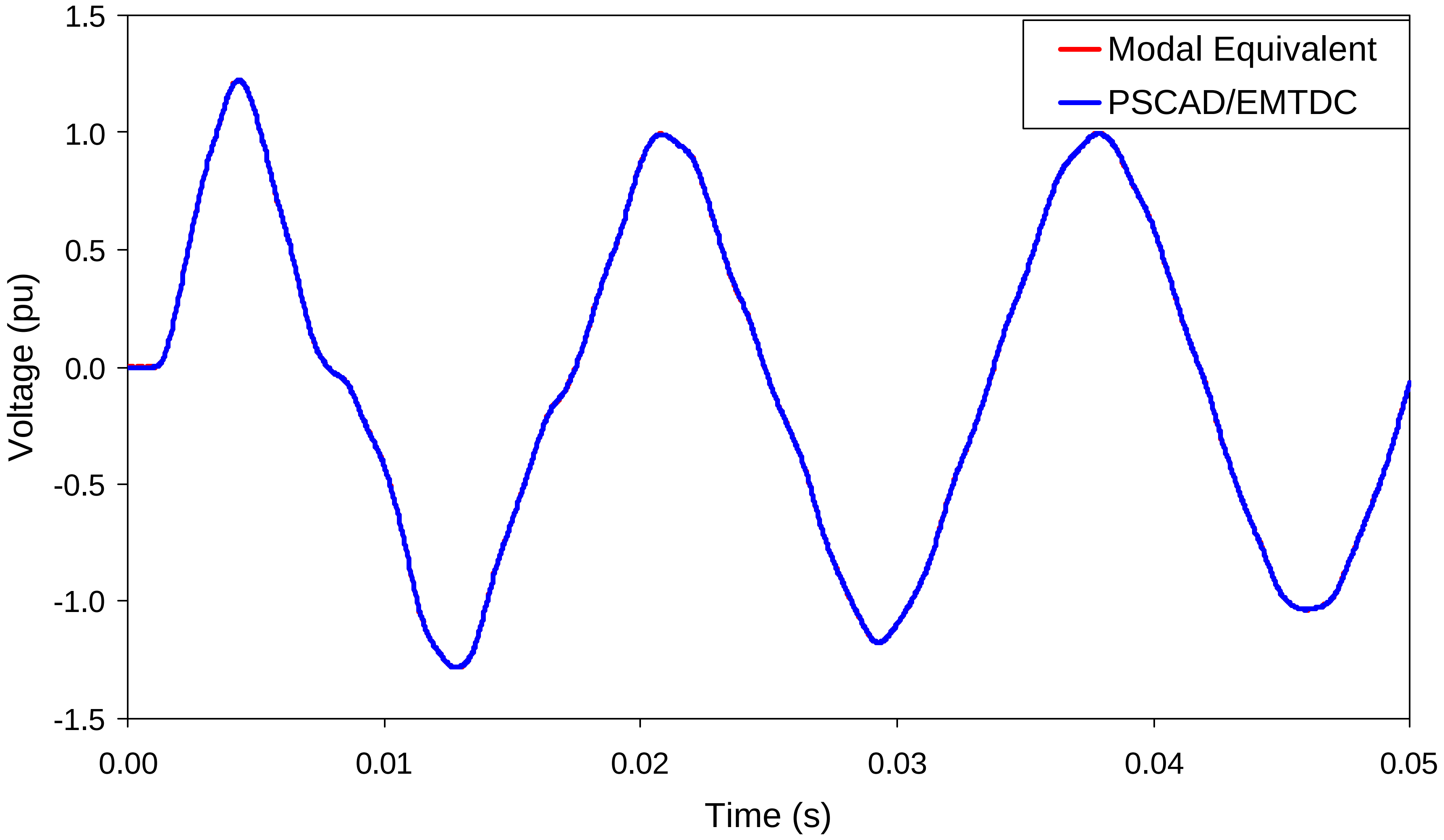
Time Response of Modal Components for Sine Input



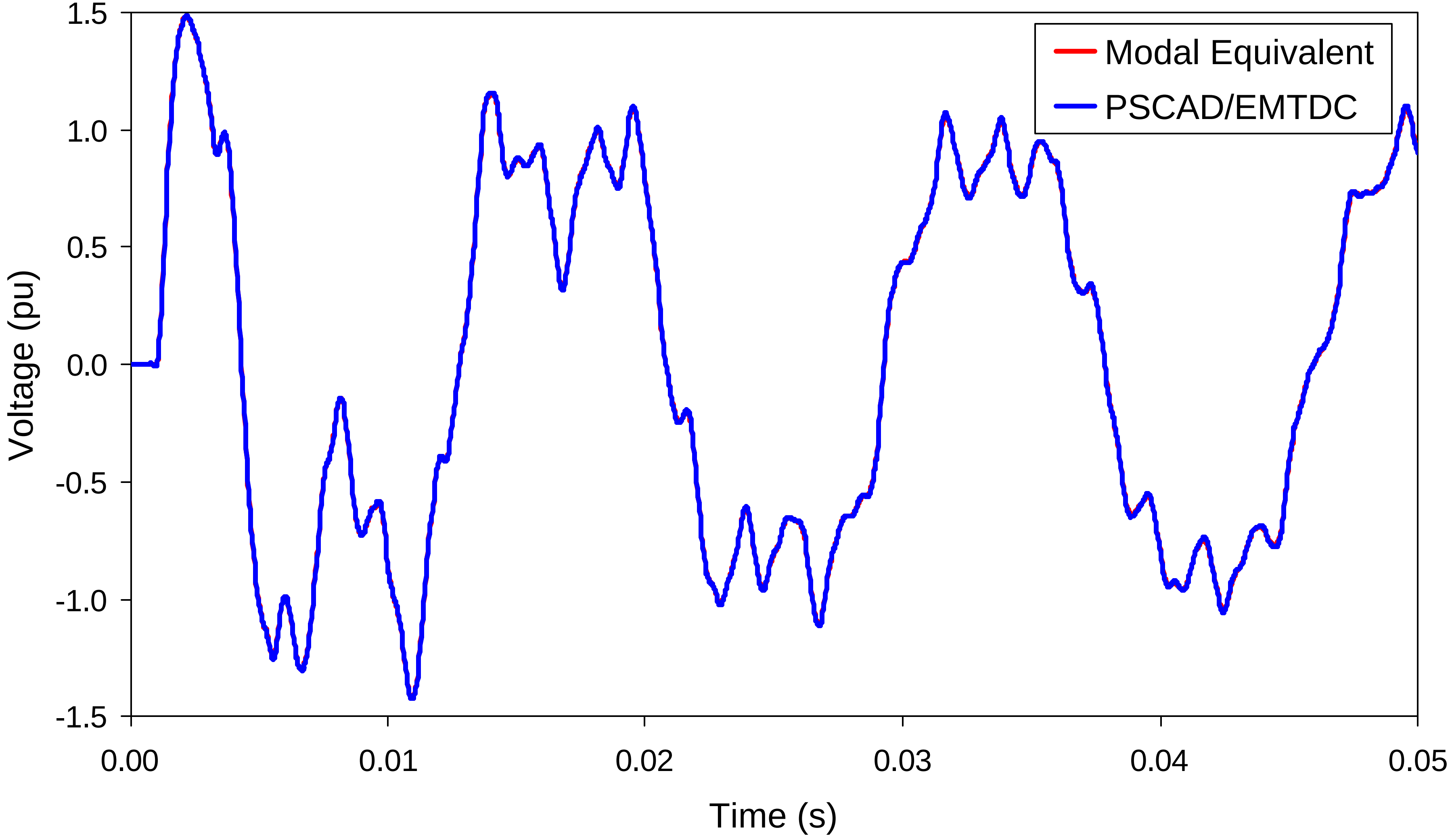
Time Response of Modal Components for Cosine Input



Comparing Modal Equivalent with PSCAD/EMTDC (Sine)



Comparing Modal Equivalent with PSCAD/EMTDC (Cosine)



Determining Maximum Overvoltage

- **Determination of the maximum overvoltage when the phase of the voltage source is varied.**

- **Simulations using Modal Equivalent and PSCAD/EMTDC.**

  - **→335 simulations**

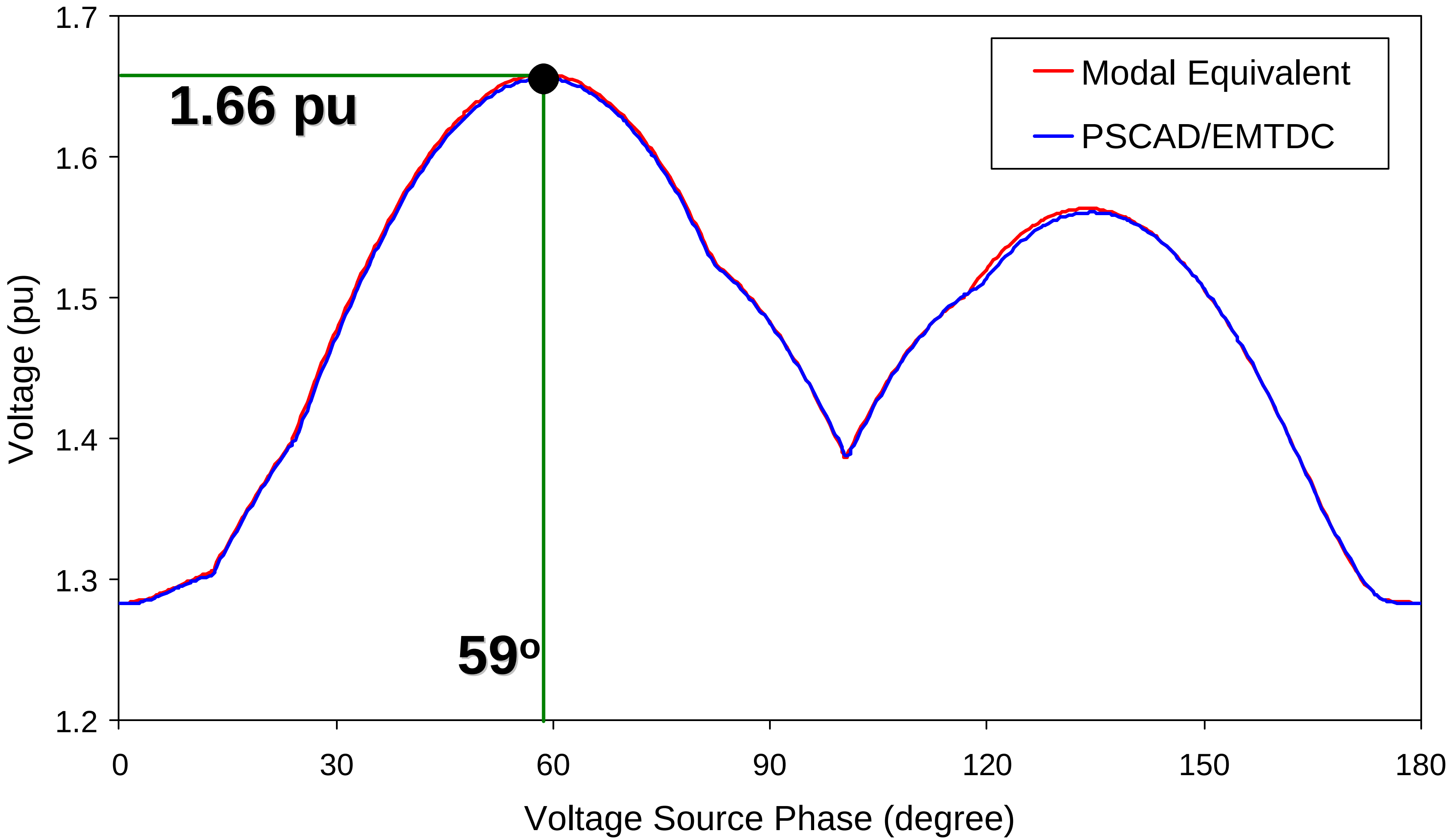
  - **→Angle varied from 0 to 180°**

  - **→Simulation time = 0.05 s**

  - **→Time step = 25 μs**

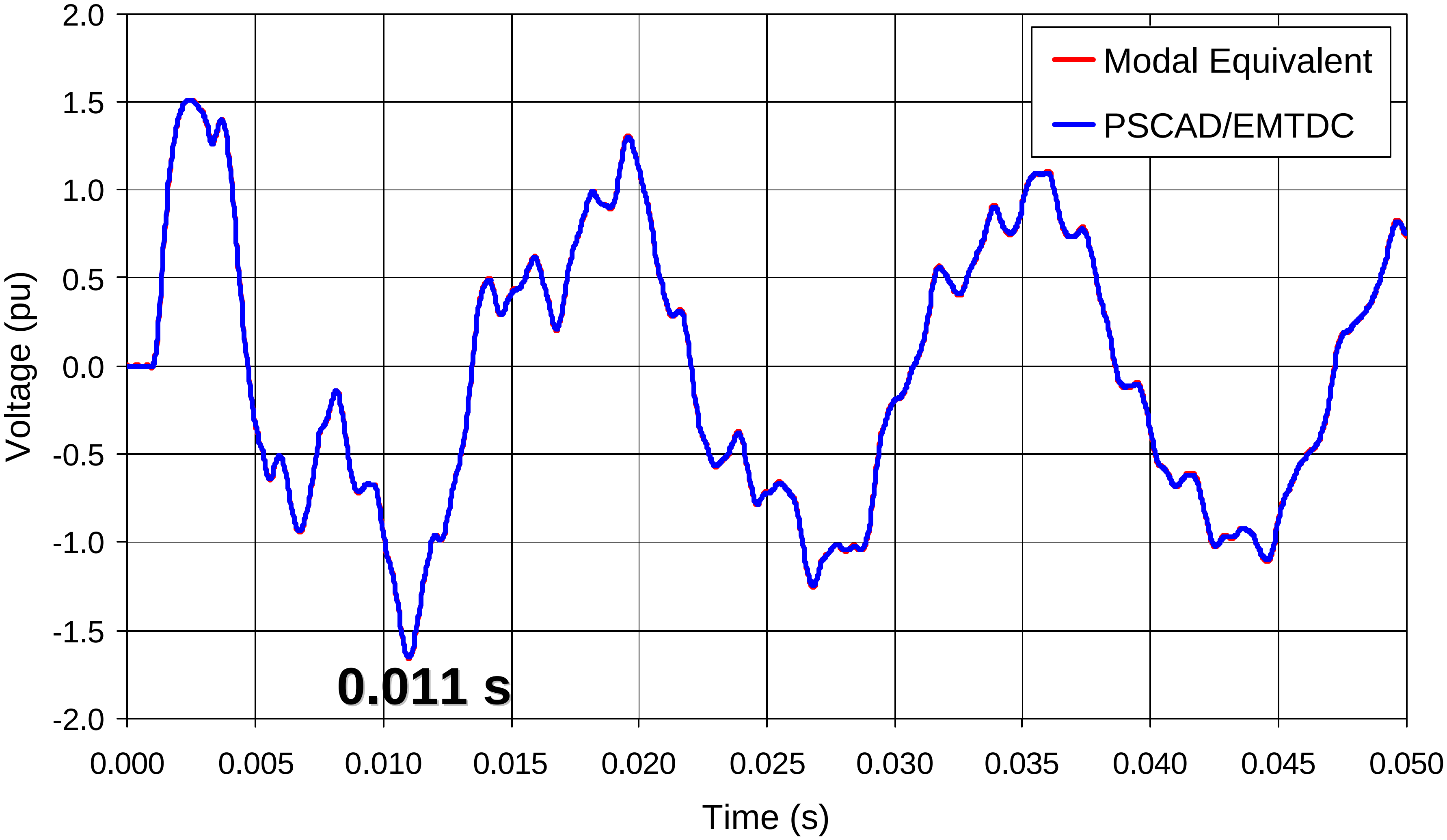
  - **→Angle step corresponding to the time step**

Maximum Overvoltage



- Modal equivalent takes 3 s and PSCAD/EMTDC takes 7 minutes in a Pentium III 500 MHz.

Case that yields the maximum overvoltage



Conclusions

- **The paper presents the basis for the use of modal analysis of electromagnetic transients in ac networks.**
- **Modal analysis of electromagnetic transients in systems having long transmission lines is best carried out using s-domain model for ac network.**
- **Modal equivalents can be used to efficiently obtain time response of electromagnetic transients.**
- **The results relates to a very simple example system, that may be reproduced by the readers. It must be pointed out that large-scale networks having transmission lines with frequency dependent parameters can also be efficiently analyzed using the proposed methodology.**
- **The modal analysis can be used to get structural information about the system and means to improve dynamic performance.**