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## IX SYMPOSIUM OF SPECIALISTS IN ELECTRIC OPERATIONAL AND EXPANSION PLANNING

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# Subsynchronous Resonance Results Obtained with a Comprehensive Small-Signal Stability Program

Sergio Gomes Jr.\*  
CEPEL

Paulo E. M. Quintão  
Fundação Padre Leonel Franca

Nelson Martins  
CEPEL

### Abstract

The paper presents a comprehensive tool for small signal stability analysis of subsynchronous resonance. Results showing the application of this tool to a benchmark system are presented. These results show the power of small signal stability analysis in providing structural information about the system, which nicely complement the conventional time domain simulation usually used in this kind of studies.

**Keywords:** Subsynchronous Resonance, Small-Signal Stability, Linear Analysis, Eigenvalue, Modal Analysis.

### 1 - INTRODUCTION

Subsynchronous resonance (SSR) is a phenomenon that occurs in thermal power plants, whose turbo-generator components (synchronous generator, rotating exciter and multi-stage steam turbines) are connected through a very long shaft. These turbo-generator components are modeled as lumped masses and have considerably high inertia constants. Shaft sections are modeled by torsional springs, whose stiffness varies with the shaft section diameter. This spring-mass system has therefore several natural modes of oscillation, known as torsional modes [1, 2].

SSR problems are due to the adverse interaction between the electrical network subsynchronous modes and the torsional modes of the turbo-generator. Electrical torques at subsynchronous frequencies occur in generators associated with heavily series-compensated transmission systems and may excite poorly-damped torsional modes to the point of causing shaft damage.

Torsional modes may also adversely interact with fast excitation control systems of large turbo-generators equipped with power system stabilizers derived from rotor speed. Torsional modes in turbo-generators may also be excited by HVDC controls of nearby converter stations [2].

SSR analysis invariably requires nonlinear time domain simulations in production-grade programs for electromagnetic studies. Numerous nonlinear time domain simulations must be carried out and experienced engineers are needed to interpret the results and infer system characteristics. Linear analysis techniques are also very much used, being complementary to nonlinear time domain simulations. They provide a large set of structural information on the system in a direct and effective way. There are many possibilities offered by linear analysis, including [3-7]:

- Identification of the dominant characteristics of the individual oscillatory modes, determining their mode shapes and associated information on node points, points of largest amplitude, coherent groups, etc.
- Determination of the critical torsional or subsynchronous modes and the parameters that most influence them. Also the identification of mitigating actions involving parameter changes and using eigenvalue sensitivity formulas with respect to system parameter changes.
- Proper location of sensors and actuators based on modal observability / controllability indices, properly designed controls though frequency domain, pole location, root-locus and eigenvalue sensitivities.
- Determination of the modal components that participate most in potentially dangerous SSR phenomena.
- Time simulation of the linearized equations, in descriptor system form, that can be computed much faster than the nonlinear simulation. The effective use of this function, while keeping in mind the inherent limitations of the linearized system analysis, can save valuable time in practical engineering studies.
- Determination of potential SSR resonances by the frequency scan methodology, which despite its limitations, is widely used in engineering studies for being of simple implementation.

This paper presents results, obtained with the SSR module of the PacDyn computer program, for the IEEE First Benchmark System [8].

The presented results help demonstrate the several program features that make it attractive for use in engineering studies. The eigenvalue and frequency response functions were validated by comparing the obtained results with those reported in [1, 2].

The time response function was validated by comparing the results obtained with the computer program and the PSCAD/EMTDC [9], considering small disturbances.

## 2 - SYSTEM MODELING

SSR phenomena involve oscillations in the 20-40 Hz range that require the dynamic modeling of the *RLC* electrical networks.

In addition to the electrical network transients, generator stator transients must be modeled (not needed for electromechanical studies) and also the torsional modes of the turbine-generator shaft [1].

Two alternative formulations are used in the SSR program module: the descriptor system approach and the direct *s*-domain modeling. Both formulations have been applied in the study of electromagnetic transients [10-13] and power system harmonic analysis [14-18].

For the SSR analysis, which involves generator models and associated controls, the formulations must be carried out in *d - q* coordinates.

The descriptor system modeling allows direct numerical integration (trapezoidal rule) to produce linear time responses. It also allows obtaining the full system eigensolution through use of a QZ eigenroutine [19].

The *Y(s)* model allows the study of very large-scale systems, having distributed parameter and frequency dependent components and generates more compact, and yet sparse, system matrices. The *Y(s)* model, however, cannot be used if the complete eigensolution or the system time response is required.

The descriptor system and the *Y(s)* models make use of a large set of numerical algorithms that turn practical the study of higher-frequency phenomena in power systems, such as SSR analysis, which is the main subject of this paper.

## 3 - THE COMPUTER PROGRAM

A module for subsynchronous resonance analysis was implemented into the PacDyn program. This module is called PacSSR and uses the same data files and databases of the electromechanical module [7].

The only differences in the data file is a new flag needed to specify the kind of simulation study required and a

new data code used for inputting the mechanical data for the individual masses of generators. Therefore, with this little additional information, the program is able to properly model multi-machine power systems for SSR studies.

This effective integration of the new module allows the use of the full set of numerical methods developed for the electromechanical analysis and described in [6]. Some additional methods were created or adapted to deal with the *Y(s)* modeling of large-scale power systems.

## 4 - FIRST BENCHMARK SYSTEM

The IEEE First Benchmark System [8] was used to validate the SSR module and the developed methodology as well as illustrate the application of the computational program. Figure 1 shows the one-line diagram of the 60 Hz, single-machine-infinite-bus system with a series compensated transmission system, frequently utilized in SSR studies and known as the "First Benchmark System". The transmission line impedances are given in per-unit of the machine MVA base (892.4 MVA).

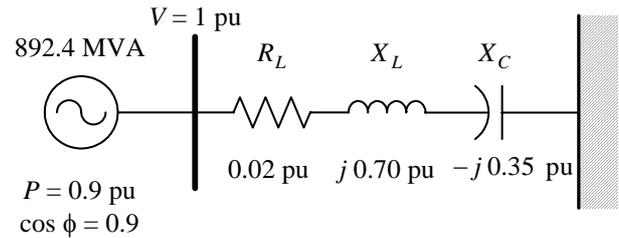


Figure 1 – IEEE First Benchmark System

The electrical generator data are shown below, in the 892.4 MVA base:

$$\begin{aligned} X_d &= 1.79 \text{ pu} & X_q &= 1.71 \text{ pu} \\ X'_d &= 0.169 \text{ pu} & X'_q &= 0.228 \text{ pu} \\ X''_d &= 0.135 \text{ pu} & X''_q &= 0.200 \text{ pu} \\ X_l &= 0.13 \text{ pu} & r &= 0 \text{ pu} \\ T'_{d0} &= 4.3 \text{ s} & T'_{q0} &= 0.85 \text{ s} \\ T''_{d0} &= 0.032 \text{ s} & T''_{q0} &= 0.05 \text{ s} \end{aligned}$$

The automatic voltage regulator (AVR) is depicted in Figure 2. It should be pointed out that the original First Benchmark System does not have an AVR.

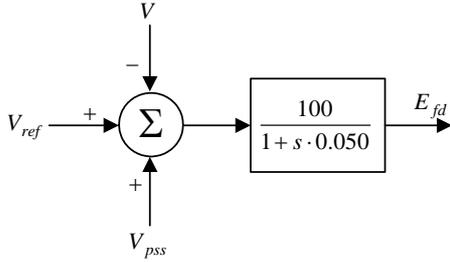


Figure 2 – Automatic Voltage Regulator

The rotating mechanical system for the generator is comprised by six masses, as shown in Figure 3:

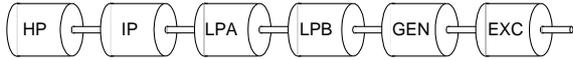


Figure 3 – Configuration of the rotating masses

The multi-mass mechanical system parameters are presented in Table 1:

Table 1 – Parameters for spring-mass system

Mass	$H$ (s)	$D$ (pu/pu)	Axis	Stiffness $K$ (pu/rad)
HP	0.092897	0.104108		
			HP-IP	19.303
IP	0.155589	0.058477		
			IP-LPA	34.929
LPA	0.858670	0.019680		
			LPA-LPB	52.038
LPB	0.884215	0.002280		
			LPB-GEN	70.858
GEN	0.868495	0.024762		
			GEN-EXC	2.822
EXC	0.0342165	0.010219		

The symbol  $H$  denotes the inertia constant for each mass,  $D$  the damping factor of each mass and  $K$  the torsional stiffness of the shaft. The mechanical damping factors have different values from those in the original benchmark system. They were calculated to produce positive damping ratios in all torsional eigenvalues when considering a 0.35 pu series capacitance reactance.

The steady-state mechanical torque is produced by the turbine sections in the following proportions: 30% (HP), 26% (IP), 22% (LPA) and 22% (LPB). The exciter steady-state torque is assumed to be zero.

## 5 - RESULTS

The results presented in this paper, show the application of the PacSSR module of the PacDyn program to the IEEE First Benchmark System.

In order to better describe the main functions of each controller and the adverse interactions it may induce, they were sequentially introduced into the IEEE First Benchmark System. AVR, power system stabilizes

(PSS) and torsional filters are sequentially introduced, and their positive and adverse impacts are described through many results (time responses, eigenvalue results, mode shapes, frequency response, root locus).

Figure 4 shows the time simulation results for a step disturbance applied to the synchronous machine mechanical power ( $\Delta P_{mec}$ ), where the monitored variable is the synchronous machine rotor speed ( $\Delta\omega$ ). The time response results obtained by the PacSSR module in PacDyn are very similar to those obtained by PSCAD/EMTDC, showing that the modeling for machine and electric network is compatible with the models used in known commercial programs for time simulation of electromagnetic transients.

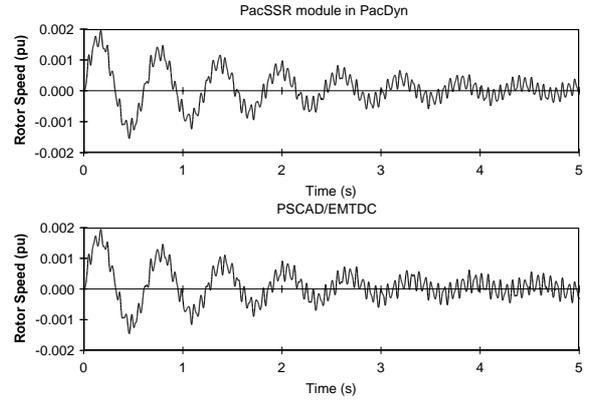


Figure 4 – Results produced by PacSSR and PSCAD

The frequency response of the transfer function where the input variable is the mechanical torque (pu) and the output variable is the generator (GEN) rotor speed (pu) is shown in Figure 5.

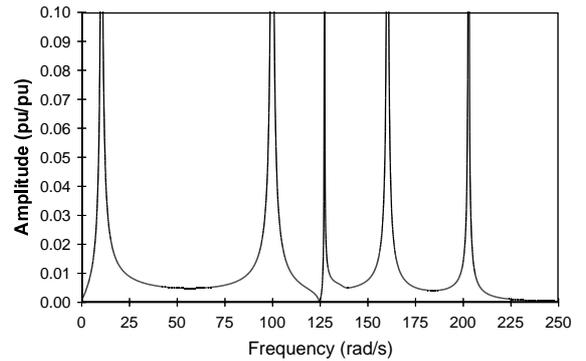


Figure 5 – Frequency response of  $\omega_g / T_{mec}$

Figure 5 shows there are dominant poles [3] in frequencies around 10 rad/s, 100 rad/s, 130 rad/s, 170 rad/s and 210 rad/s. These (purely imaginary) estimates were used in the Dominant Pole Spectrum Eigensolution (DPSE) algorithm [4], which converged to the following five complex eigenvalues:

Table 2 – Transfer function dominant poles computed by DPSE (system without AVR)

Transfer Function Dominant Poles Computed by DPSE	Frequency (Hz)	Damping (%)
$-0.01506 + j 202.79$	32.27	0.01
$-0.12076 + j 160.28$	25.51	0.08
$-0.00061 + j 127.23$	20.24	0.00
$-0.00043 + j 99.797$	15.88	0.00
$-0.48770 + j 10.286$	1.64	4.72

The five complex-conjugate pairs may be used to obtain a reduced order model [3,4]. The comparison between the full model and this reduced order model in the frequency domain is presented in Figure 6. The curves are visually coincident over the 0-40 Hz frequency range.

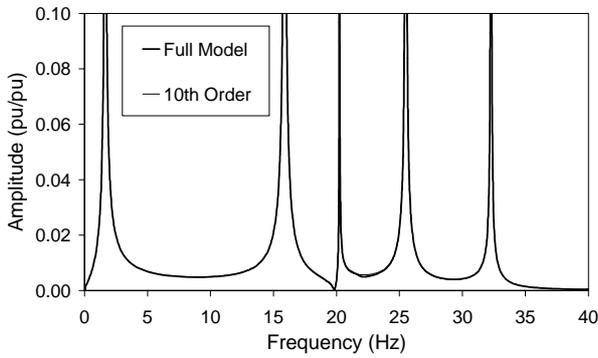


Figure 6 – Comparison between the frequency responses of  $\omega_g / T_{mec}$  considering a 10<sup>th</sup> order reduced model and the full model.

The time responses for the speed deviations of the various masses are shown in Figure 7. The transfer function input utilized is the mechanical torque ( $T_{mec}$ ) and the output variables are the speeds of the various masses. The generator active power deviations are shown in Figure 8.

It should be noted that the electromechanical mode is fairly damped (damping ratio of 4.72%) but the torsional modes are very low damped (about 0%), so there are sustained oscillations of relatively small amplitude.

The full eigensolution of the system was obtained by using the QZ method. A high participation factor was the index used to identify the nature of each mode, as indicated in the third column of Table 3.

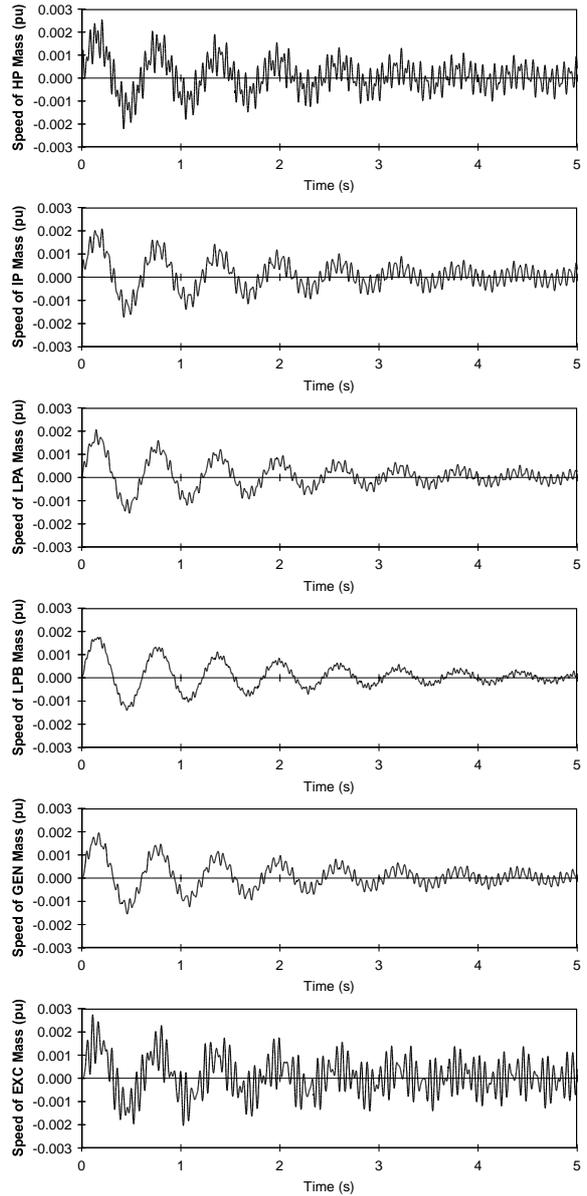


Figure 7 – Time response of the speeds of the various masses following a step of 0.1 pu in the mechanical torque  $T_{mec}$

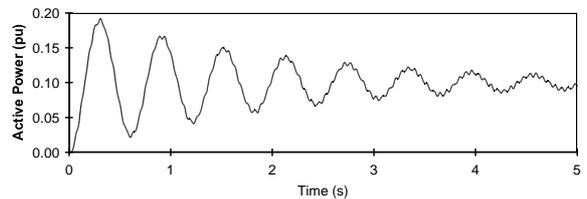


Figure 8 – Time response of the terminal active power for a step of 0.1 pu in the mechanical torque  $T_{mec}$

Table 3 – System poles without AVR – SSR model

Poles without AVR	Freq. (Hz)	Description
$-4.6337 + j 616.62$	98.14	Supersynchronous
$-3.3300 + j 136.79$	21.77	Subsynchronous
$-0.1379 + j 298.18$	47.46	Torsional
$-0.0151 + j 202.79$	32.27	Torsional
$-0.0121 + j 160.28$	25.51	Torsional
$-0.0006 + j 127.23$	20.24	Torsional
$-0.0004 + j 99.797$	15.88	Torsional
$-0.4877 + j 10.319$	1.64	Electromechanical
-41.198	0	Generator
-25.425	0	Generator
-3.0734	0	Generator
-0.2486	0	Generator

This eigenvalue list produced when using the SSR model (Table 3) should be compared with the eigenvalue list obtained using the model for electromechanical stability analysis (Table 4). There is good agreement for the low frequency eigenvalues produced by these two modeling levels.

Table 4 – System Poles without AVR – Electrom. Model

Poles without AVR	Freq. (Hz)	Description
$-0.4971 + j 10.378$	1.65	Electromechanical
-41.258	0	Generator
-25.426	0	Generator
-3.0735	0	Generator
-0.2487	0	Generator

The speed mode-shapes of each rotating mass carry information on the nature of each oscillatory mode (either torsional or electromechanical). The mode-shapes of these modes are shown in Figure 9. It should be noted that for the electromechanical mode (1.64 Hz), all masses oscillate together. Each one of the five torsional modes (between 15.9 Hz and 47.5 Hz) has a specific characteristic or “shape”. For example, regarding the lowest frequency torsional mode, the three masses on the left side oscillate at 15.9 Hz against the three masses on the right side.

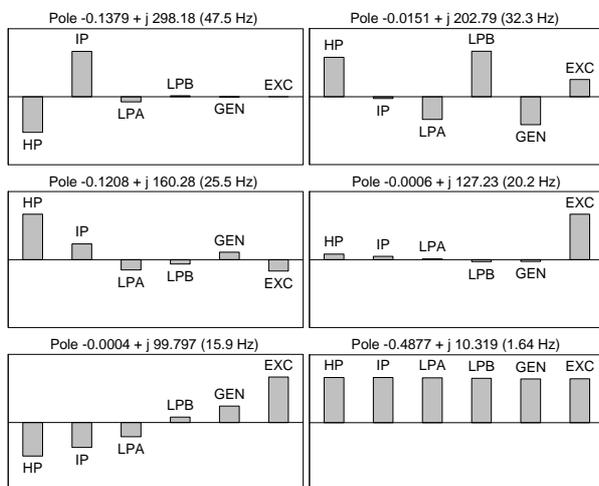


Figure 9 – Speed Mode-shapes of the mechanical masses

The value initially selected ( $X_C = 0.35$  pu) for the series capacitor reactance did not produce subsynchronous resonance phenomena because all torsional modes have positive damping ratios (all poles with negative real parts). Using the Root Locus function in PacDyn it is possible to identify the  $X_C$  values that could produce subsynchronous resonance.

Figure 10 shows the Root Locus plot obtained when varying the series capacitor reactance  $X_C$ . There is a root locus branch representing the network subsynchronous mode and another four branches representing the torsional modes of the rotating masses.

As the reactance  $X_C$  is increased, the frequency of subsynchronous network mode is reduced, while the frequency of supersynchronous network mode is increased. Whenever the frequency of the network subsynchronous mode approaches the frequency of a torsional mode, they strongly interact: the net effect is that the subsynchronous pole shifts to the left while the torsional pole shifts to the right and the system becomes unstable. This adverse dynamic interaction is known as subsynchronous resonance phenomena. The critical values of the reactance  $X_C$ , which cause the maximum shifts of the torsional modes to the right, are shown in Figure 10 and in Table 5. Figure 11 is an enlarged view of Figure 10 for the most severe resonance.

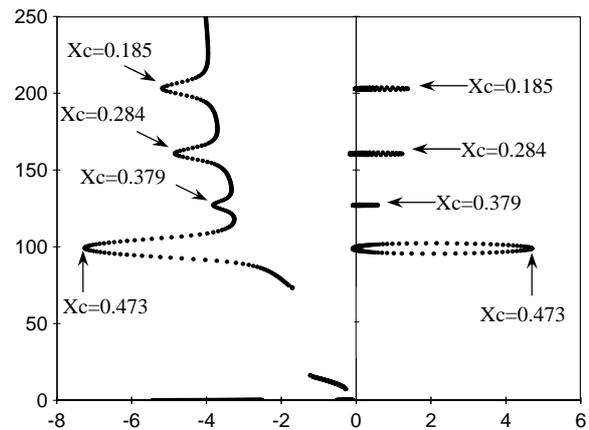
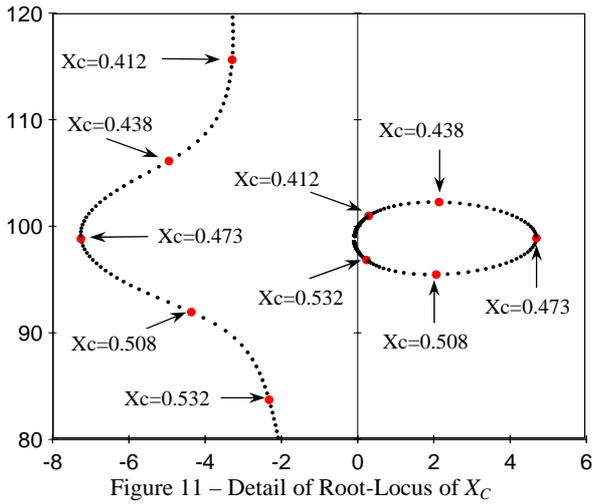


Figure 10 – Root-Locus of  $X_C$

Table 5 – Critical values of series capacitance for SSR

Capacitor Reactance (pu)	Pole	Frequency (Hz)
0.1846	$1.3854 + j 202.83$	32.3
0.2844	$1.2311 + j 160.58$	25.5
0.3786	$0.5808 + j 127.00$	20.2
0.4728	$4.7012 + j 98.84$	15.7



The linear time domain simulation for this critical value of series capacitor reactance ( $X_C = 0.473$  pu) is shown in Figure 12 and indicates oscillatory instability due to subsynchronous resonance at 15 Hz. The applied disturbance was a step of 0.1 pu in the mechanical torque of the generator and the monitored variable was the generator speed.

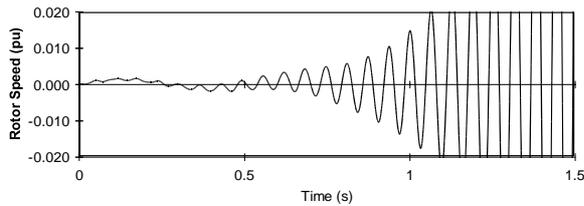


Figure 12 – Linear time response for  $X_C = 0.473$  pu

Figure 13 shows the same plot as Figure 12, the difference being that the vertical scale is now enlarged. The same simulation was then performed in the PSCAD/EMTDC program, and the result obtained is presented in Figure 14. A comparison between Figure 13 and Figure 14 shows that the linear model is quite accurate to detect small-signal subsynchronous resonance.

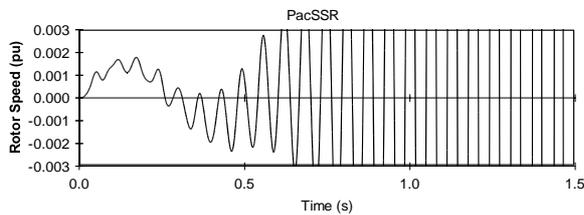


Figure 13 – Detail of linear time response for  $X_C = 0.473$  pu

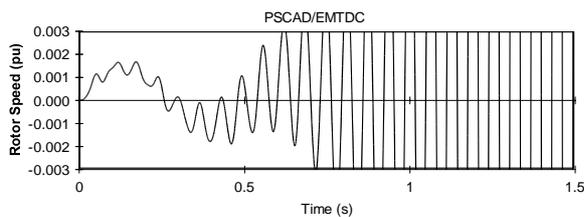


Figure 14 – Non-linear time response for  $X_C = 0.473$  pu

The AVR was then included, considering the  $X_C$  reactance equal to  $-0.35$  pu. The full eigensolution of the system obtained using the QZ method for this case is given in Table 6.

Table 6 – Poles for SSR model with AVR

Poles with AVR	Freq. (Hz)	Description
$-4.6330 + j 616.63$	98.14	Supersynchronous
$-3.2791 + j 136.78$	21.77	Subsynchronous
$-0.1379 + j 298.18$	47.46	Torsional
$-0.0151 + j 202.79$	32.27	Torsional
$-0.1209 + j 160.28$	25.51	Torsional
$-0.0021 + j 127.23$	20.25	Torsional
$-0.0063 + j 99.797$	15.88	Torsional
$-8.7594 + j 11.650$	1.85	Exciter
$-0.0043 + j 10.287$	1.64	Electromechanical
$-45.941$	0	Generator
$-25.417$	0	Generator
$-2.2284$	0	Generator

Comparing the results obtained for the case with and without AVR, one should note that the AVR has a low interaction with the torsional modes and this interaction is positive, yielding a little extra damping to some of these modes. On the other hand, the AVR adversely interacts with the electromechanical mode causing negative damping torque [2]. The electromechanical mode has now become very poorly damped. Figure 15 shows the time response of the rotor speed, for a 0.1 pu step in the mechanical torque. The result shows sustained oscillations due to the very poorly damped electromechanical mode.

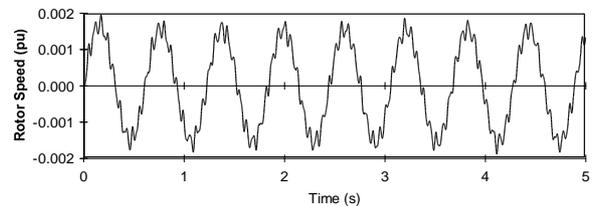


Figure 15 – Time response of SSR Model with AVR

The eigenvalue results obtained when using the SSR model (Table 6) are compared with those obtained for the electromechanical stability model (Table 7). This comparison shows there is a good agreement of the low frequency eigenvalues obtained by the two models.

Table 7 – All Poles with AVR – Electromechanical Stability Model

Poles without AVR	Freq. (Hz)	Description
$-8.3369 + j 11.414$	1.82	Exciter
$-0.0329 + j 10.595$	1.69	Electromechanical
$-46.607$	0	Generator
$-25.425$	0	Generator
$-2.2295$	0	Generator

The same simulation was then repeated for the electromechanical model and the results are shown in Figure 16.

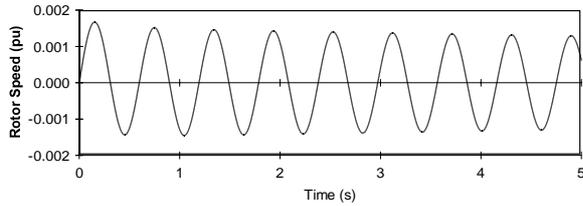


Figure 16 – Time response of Electromechanical Stability Model with AVR

Despite causing negative damping torque, the AVR is needed to promote fast voltage regulation. Before AVR inclusion the mode associated with the generator voltage dynamics was  $-0.2486$ , which corresponded to a time constant of 4 s. Now it is  $-2.2284$ , which corresponds to a time constant of 0.45 s. Figure 17 shows the voltage of generator after the switching of a 10 Mvar reactor at the generator bus. In the case without AVR the voltage drops and remains at a low value, but in the presence of AVR the voltage recovers pretty fast. There is, however, a low-amplitude, sustained oscillation due to the electromechanical mode. The electromechanical model was used, but similar results would be obtained when using the SSR model.

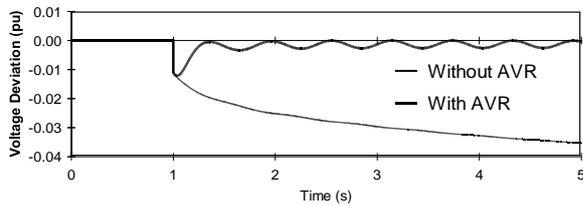


Figure 17 – Time response of the Voltage for the cases with and without AVR

A value of 0.35 pu for  $X_C$ , which does not cause subsynchronous resonance was used to design a power system stabilizer (PSS) to further damp the electromechanical mode (see Figure 18):

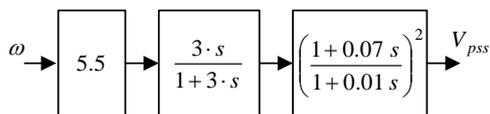


Figure 18 – Power System Stabilizer (PSS)

The system eigenvalues for  $X_C = 0.35$  pu and in the presence of the designed PSS, are listed in Table 8. The electromechanical mode was damped by the PSS, but two torsional modes were shifted to the right, causing the instability. The torsional instabilities are due to adverse interactions induced by the PSS at higher system frequencies.

Table 8 – All Poles with PSS – SSR Model

Poles	Freq. (Hz)	Description
$-4.6311 + j 616.63$	98.14	Supersynchronous
$-3.4303 + j 136.55$	21.73	Subsynchronous
$-0.1378 + j 298.18$	47.46	Torsional
$+0.0048 + j 202.75$	32.27	Torsional
$-0.1074 + j 160.19$	25.49	Torsional
$+0.0705 + j 127.32$	20.26	Torsional
$+0.2175 + j 100.30$	15.96	Torsional
$-7.0202 + j 12.784$	2.03	Exciter
$-2.1476 + j 10.338$	1.65	Electromechanical.
$-101.73 + j 10.471$	1.67	PSS
$-42.069$	0	Generator
$-25.416$	0	Generator
$-2.1632$	0	Generator
$-0.3375$	0	PSS

The time response of the generator speed for a step in the mechanical torque is shown in Figure 19. The system is unstable.

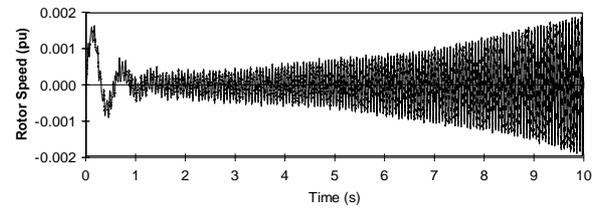


Figure 19 – Time response of SSR Model with PSS

In order to avoid adverse torsional interaction due to PSS action, the PSS input signal must have low observability of the torsional modes. This proper signal conditioning may be obtained by applying a torsional filter to the speed signal. This filter avoids the shifts of the torsional modes to the right when the PSS loop is closed. The torsional filter utilized is pictured in Figure 20. The filter parameters were chosen to produce a zero in the 15 Hz (minimum frequency to appear a torsional mode) and so that the poles needed for the filtering, did not produce a high gain in the electromechanical frequency range, in order to avoid system instability [2].

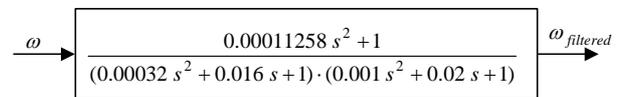


Figure 20 – Torsional Filter for PSS

The frequency response of the filter is shown in Figure 21. The torsional filter gain remains below 0.015 pu for frequencies higher than 100 rad/s (15.9 Hz), which explains the apparently zero value for the filter amplitude at this frequency range.

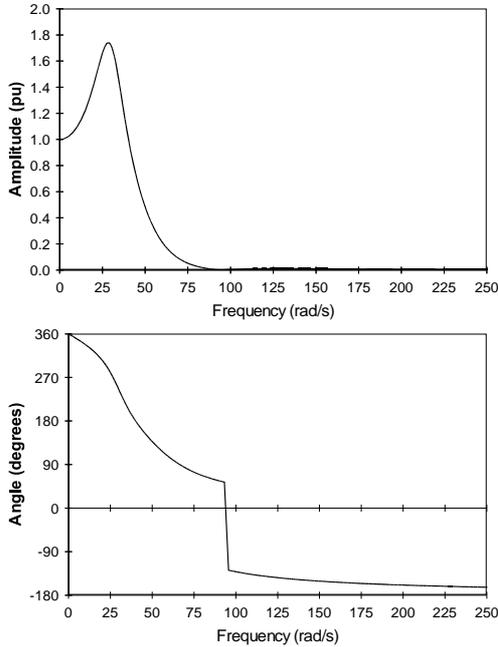


Figure 21 – Frequency Response of the Torsional Filter

The simulation of the filtered speed, before the closing of the PSS loop (using a zero gain after the filter), is shown in Figure 22. The filtered signal, for practical purposes, does not have torsional components. The comparison with the non-filtered speed is shown in Figure 23.

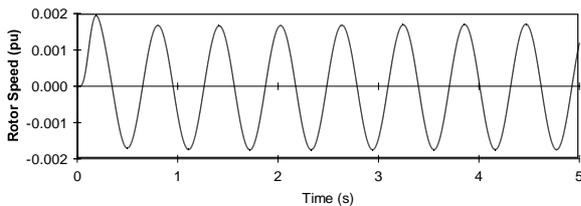


Figure 22 – Time Response of the Filtered Rotor Speed

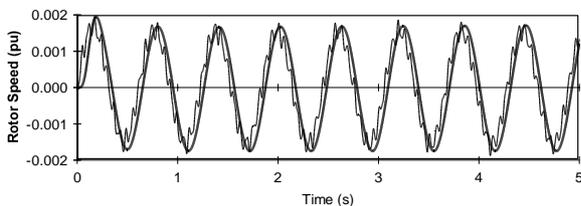


Figure 23 – Comparison between filtered and non-filtered Time Response of Rotor Speed

The filter introduced a moderate amplification and lag to the non-filtered speed signal. The parameters of the PSS may be adjusted to compensate for these changes.

The gain was reduced from 5.5 to 3.6 and the time constant of the lead block numerator was increased from 0.073 to 0.094. These parameters were adjusted so as to maintain the same damping ratio for the electromechanical mode.

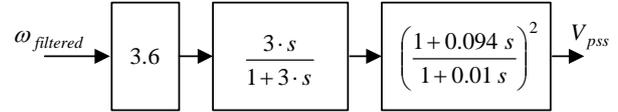


Figure 24 – PSS for Filtered Rotor Speed

The system eigenvalues considering the PSS with the torsional filter are listed in Table 9. The torsional filter effectively minimized the adverse interaction among the PSS and torsional masses, since it damped the electromechanical mode (damping ratio of 20.1%), without significantly shifting the torsional modes to the right. The exciter mode became slightly less damped but still sufficiently high (damping ratio was reduced from 48.2% to 39.8%) due to the higher loop gain around 2 Hz.

The time response of the generator speed for the same 0.1 pu step in mechanical torque is shown in Figure 25. The system is stable with a damped electromechanical mode and sustained torsional oscillations at 15 and 20 Hz. These oscillations are compatible with the utilized damping constants. Comparing tables 3, 6 and 9, one can see that the PSS with the torsional filter did not cause negative damping torques at the torsional mode frequencies.

Table 9 – All Poles with PSS and Torsional Filter – SSR Model

Poles	Freq. (Hz)	Description
$-4.6330 + j 616.63$	98.14	Supersynchronous.
$-3.2793 + j 136.78$	21.77	Subsynchronous
$-0.1379 + j 298.18$	47.46	Torsional
$-0.0154 + j 202.79$	32.28	Torsional
$-0.1216 + j 160.28$	25.51	Torsional
$-0.00201 + j 127.23$	20.25	Torsional
$-0.00474 + j 99.795$	15.88	Torsional
$-6.2244 + j 14.344$	2.28	Exciter
$-2.1425 + j 10.334$	1.65	Electromechanical
$-12.370 + j 27.820$	4.43	Filter
$-24.670 + j 49.470$	7.87	Filter
$-100.41 + j 3.6784$	0.59	PSS
$-41.855$	0	Generator
$-25.414$	0	Generator
$-2.1882$	0	Generator
$-0.3360$	0	PSS

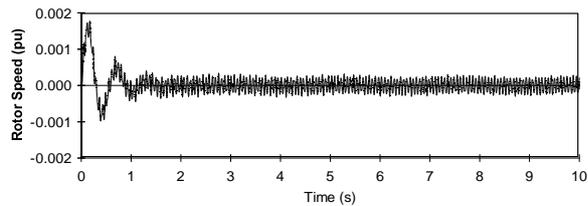


Figure 25 – Time response including PSS with torsional filter

## 6 - Conclusions

A SSR module was implemented in a comprehensive tool for small signal analysis. Subsynchronous resonance studies were carried out on the IEEE First Benchmark System, showing the advantages of using the linear analysis in obtaining structural information about subsynchronous resonance. The modeling used in the computational program produced equivalent results to those obtained with production-grade software for electromagnetic transient time simulation, which is generally employed in these studies. The presented results helped emphasize the importance of using linear techniques in SSR analysis.

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