Utilizing Transfer Function Modal Equivalents of Low-Order for the Design of Power Oscillation Damping Controllers in Large Power Systems

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Optimal Model Reduction by Balanced Truncation

Model Error: $e = \left\| G(s) - G_r(s) \right\|_\infty$

Upper Bound Error:

$$e_1 = 2 \sum_r s_i$$

s_i – Hankel singular values

n – full model order

r – reduced model order

Drawbacks:

- does not preserve original pole-zero spectrum
- pre-conditioning to deal with unstable poles
- high computational cost

Modal Equivalents

Model Error: $e = \left\| G(s) - G_r(s) \right\|_\infty$

Upper Bound Error:

$$e_2 = \sum_r \frac{C_i B_i}{\mathrm{Re}\{ l_i \}}$$

n – full model order

r – reduced model order

Characteristics:

• Reduced models built with TF dominant poles and associated residues.

•May be applied to large scale systems if efficient TF dominant pole routines are available.

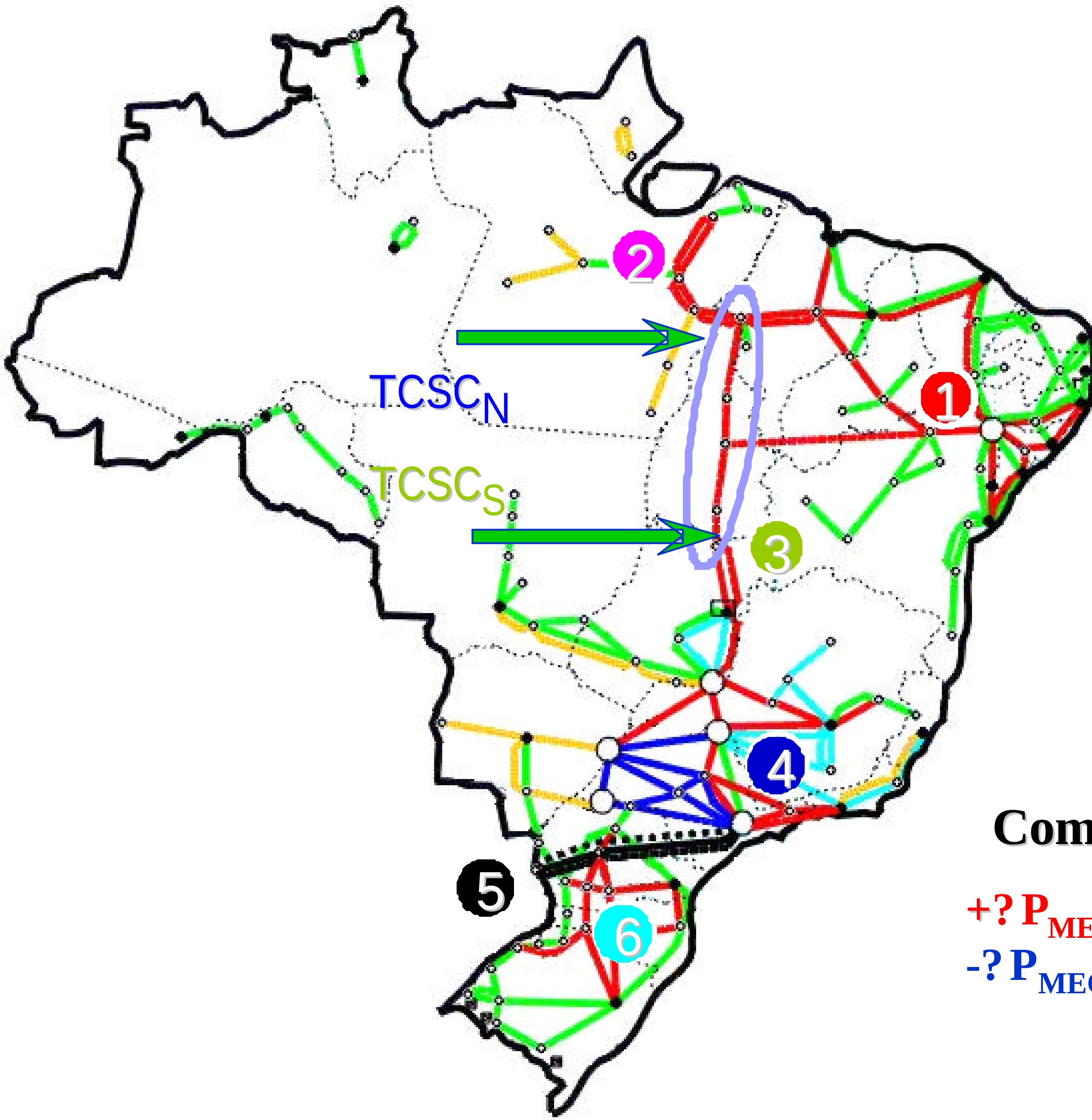
•TF zeros of the original model are not preserved.

•Good reduced models are not of minimal order.

$$\boxed{e_1 < e_2}$$

*Proof: see e.g. M.Green and D.Limebeer, Linear Robust Control, Prentice Hall, 1995.

Brazilian North-South Interconnection – with PODs



Power Plants

1 – Northeast
2 - Tucuruí
3 – Serra da Mesa
4 – Southeast
5 – Itaipu
6 – South

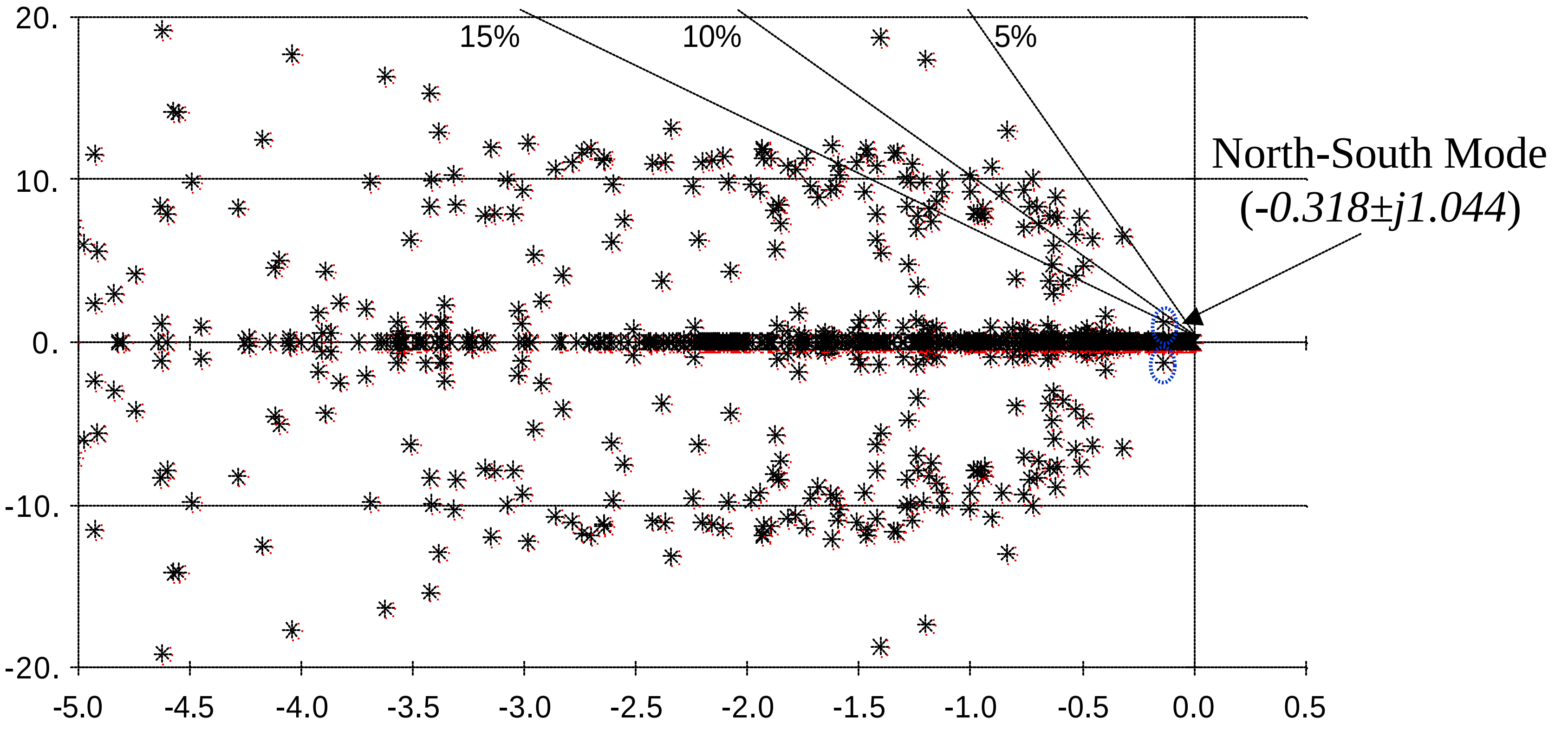
Monitored Variable

Generator Rotor Speed

Combined Input Disturbance

+? P_{MEC}^{N} at 3 Northeast Plants
-? P_{MEC}^{S} at 2 Southest Plants

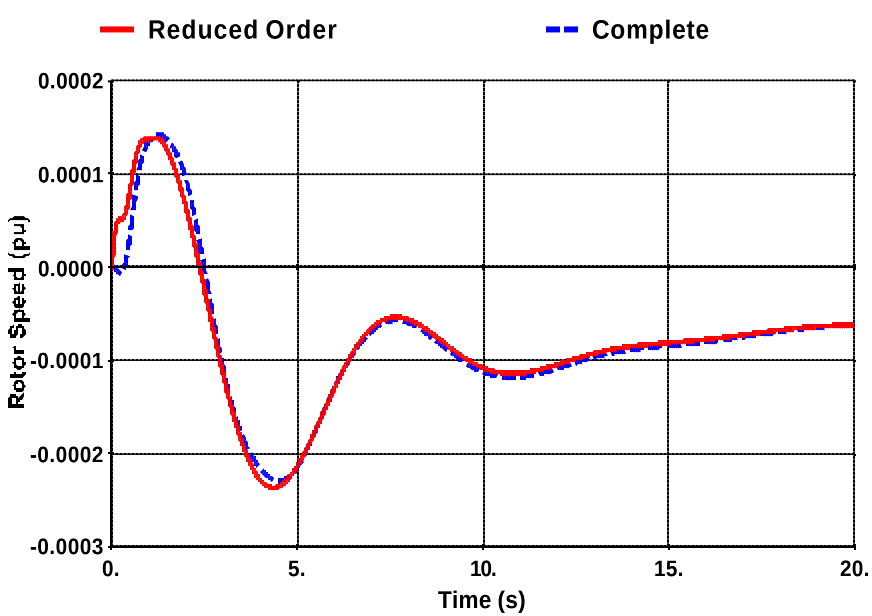
Eigenvalue Spectrum of Brazilian System Operational Planning Model – year 1999 (1,676 states)



The two TCSCs of the North-South Intertie have PODs to confer damping to the N-S mode.

Step Response of Scalar Transfer Function, having 1676 states, and its 10th order Modal Equivalent

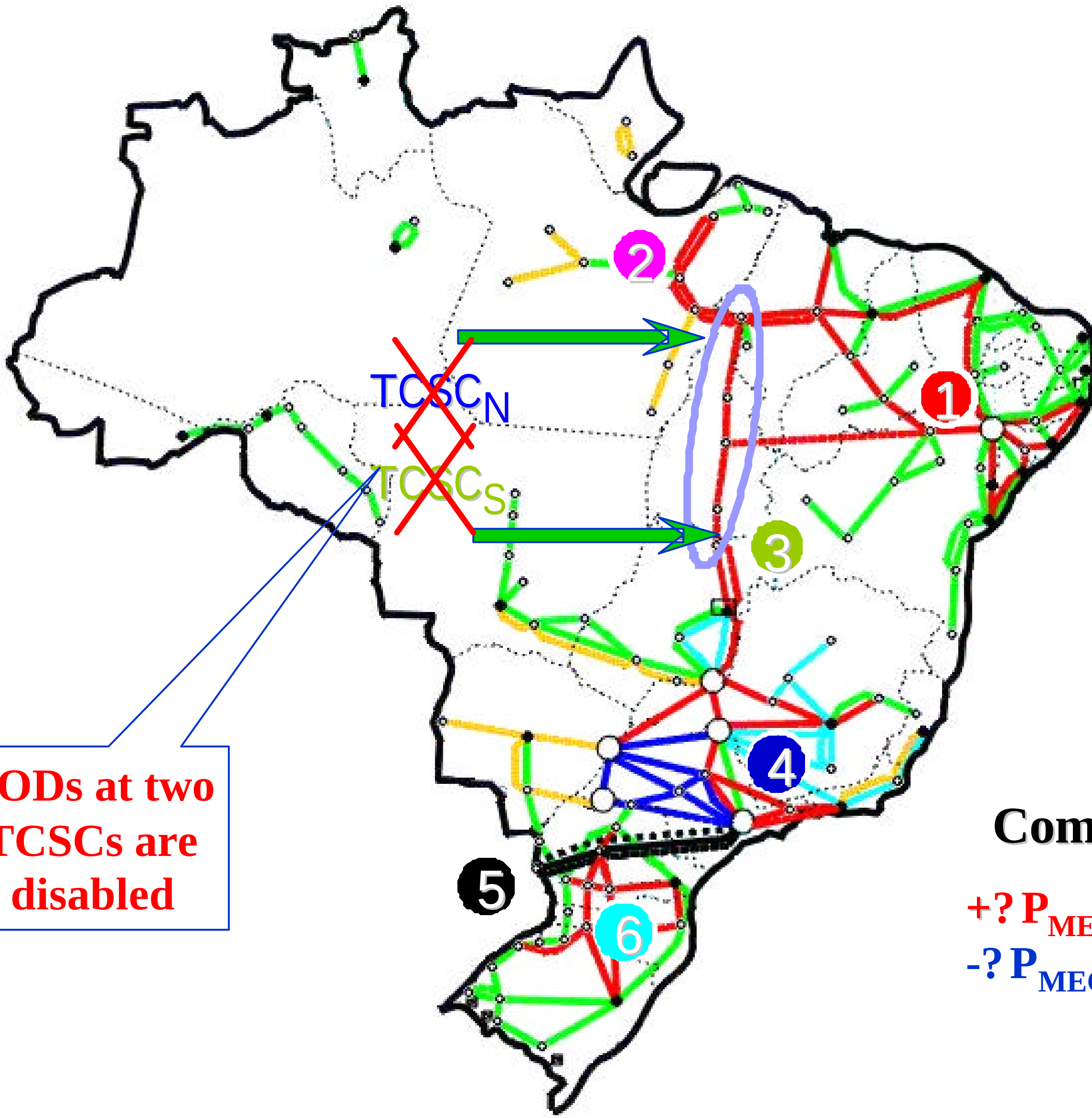
$$y(t) \cong \sum_{i=1}^{10} \frac{R_i}{l_i}\left(e^{l_i \cdot t} - 1\right)$$



Generator #5022

POLES	RESIDUES
$l_{1,2} = -7.016 \pm 2.918j$	$R_{1,2} = 0.064 \angle \pm 93°$
$l_{3,4} = -2.996 \pm 9.390j$	$R_{3,4} = 0.030 \angle \pm 44°$
$l_{5,6} = -0.318 \pm 1.044j$	$R_{5,6} = 0.022 \angle \pm 7°$
$l_{7,8} = -0.346 \pm 0.580j$	$R_{7,8} = 0.013 \angle \pm 175°$
$l_{9,10} = -0.116 \pm 0.245j$	$R_{9,10} = 0.002 \angle \pm 148°$

Brazilian North-South Interconnection – without PODs



PODs at two TCSCs are disabled

Power Plants

1 – Northeast
2 - Tucuruí
3 – Serra da Mesa
4 – Southeast
5 – Itaipu
6 – South

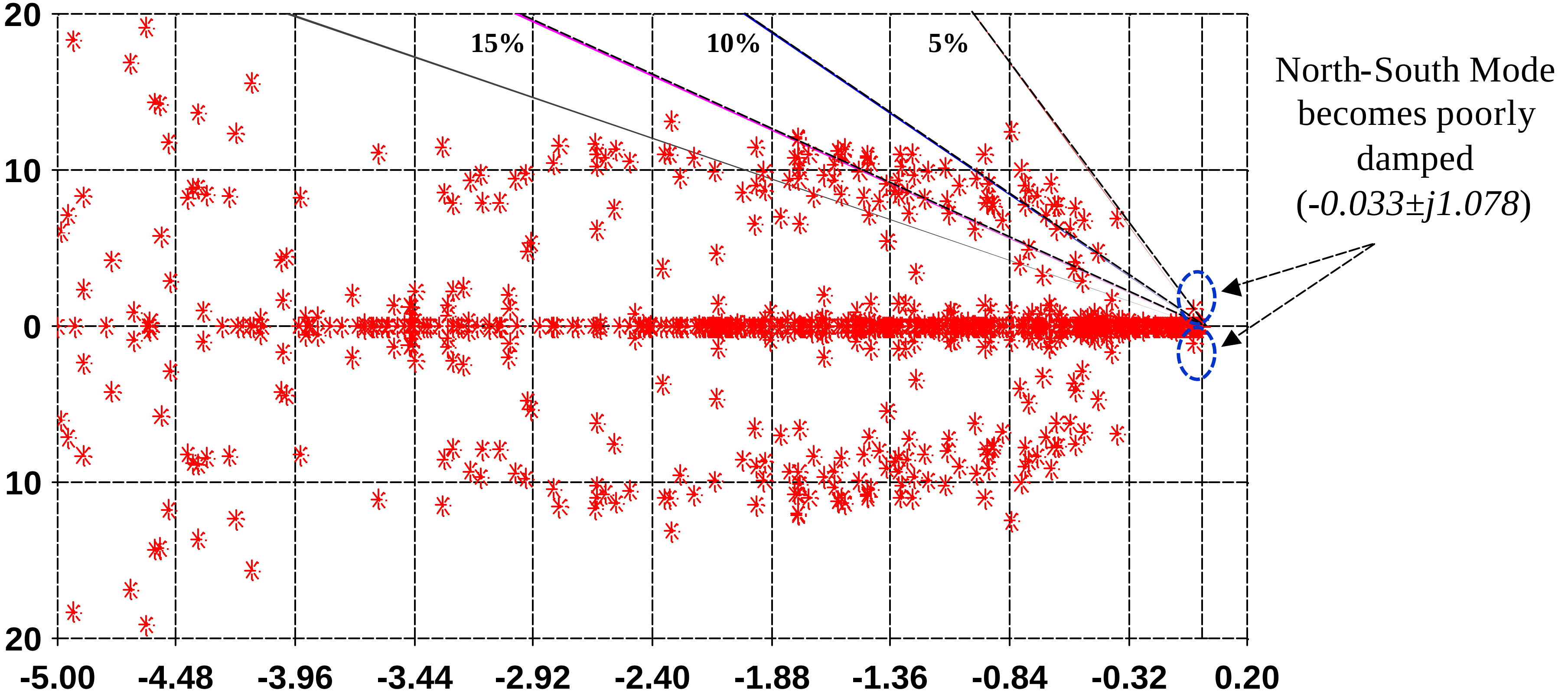
Monitored Variable

Generator Rotor Speed

Combined Disturbance

+? P_{MEC}^{N} at 3 Northeast Plants
-? P_{MEC}^{S} at 2 Southest Plants

Eigenvalue Spectrum of Brazilian System
Modified Planning Model – year 1999 (1,664 states)

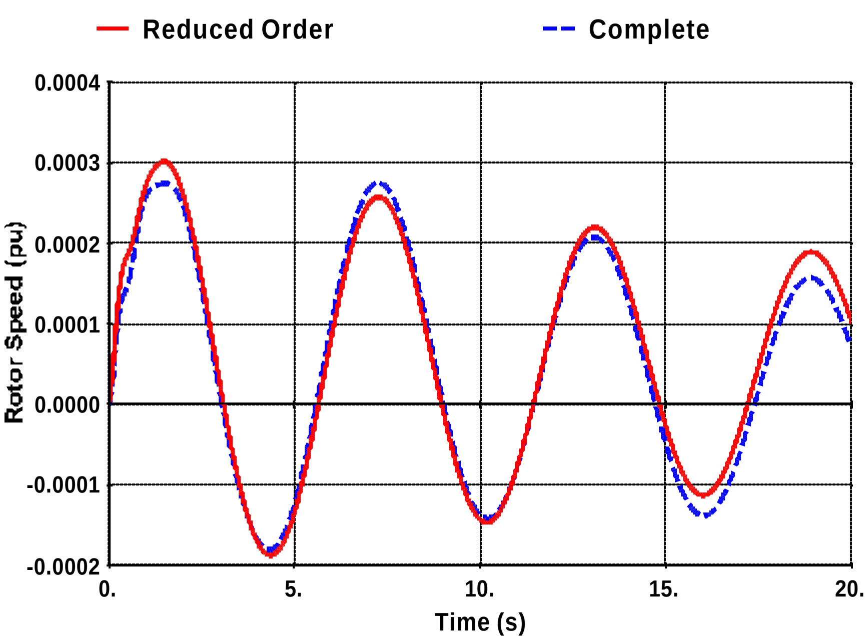


System without PODs (12 states) in the two TCSCs of the North-South Intertie

Step Response of Scalar Transfer Function, having 1,664 states, and its 6th order Modal Equivalent

$$y(t) \cong \sum_{i=1}^{6} \frac{R_i}{l_i}\left(e^{l_i \cdot t} - 1\right)$$

POLES	RESIDUES
$l_{1,2} = -0.034 \pm 1.079j$	$R_{1,2} = 0.015\angle \pm 1.3^{\circ}$
$l_{3,4} = -2.437 \pm 0.054j$	$R_{3,4} = 0.010\angle \pm 77^{\circ}$
$l_{5,6} = -0.521 \pm 2.881j$	$R_{5,6} = 0.001\angle \pm 111^{\circ}$



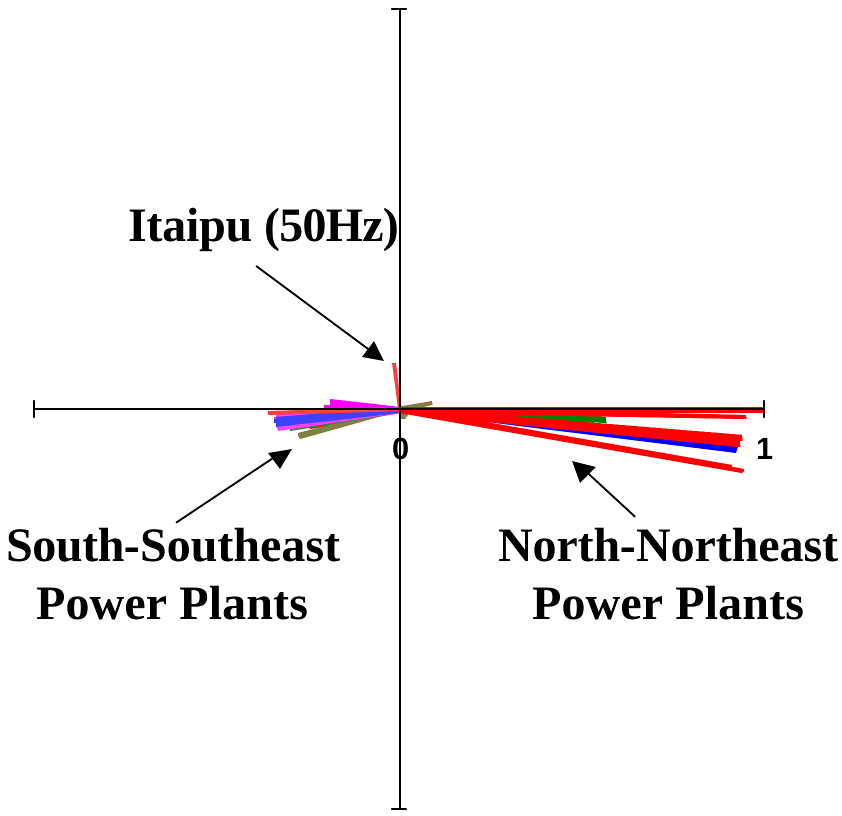
Generator #5022

Note: system without two PODs (12 states) of the North-South Interconnection

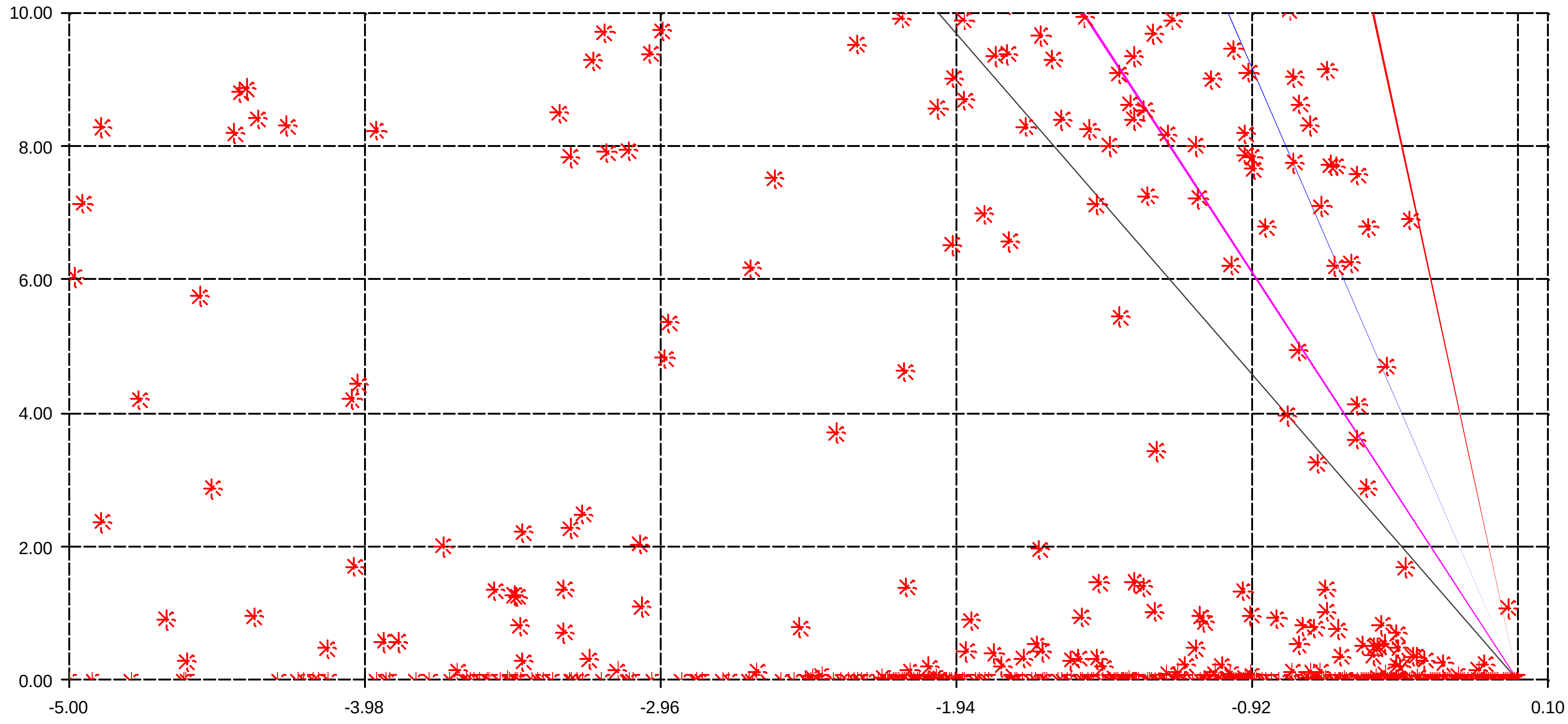
Rotor Speed Mode-Shape for North-South Mode

Output: WW
Eigen: -0.033534 +J1.0787

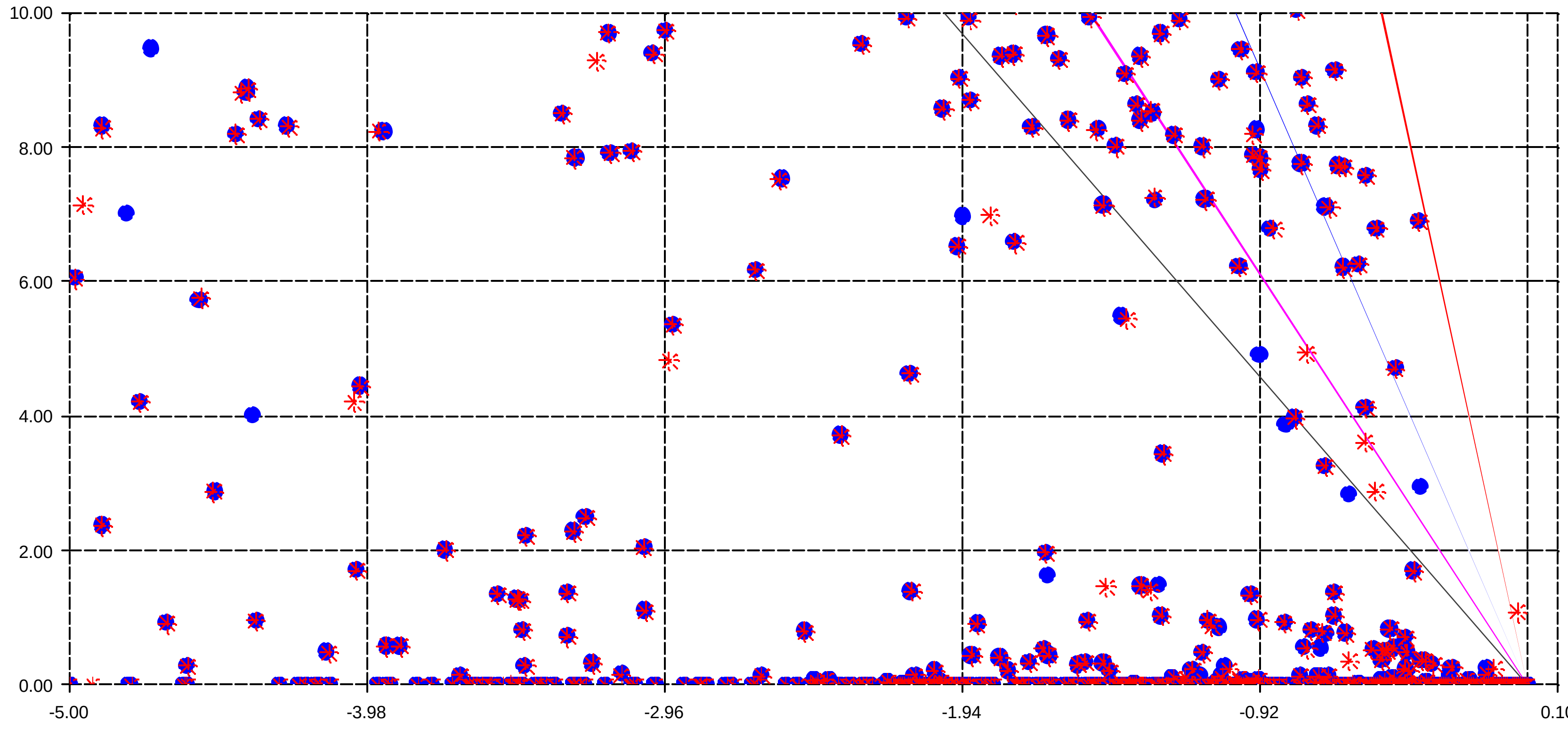
- *** COMPLEXO PAF + UAS ***
- *** AREA LESTE ***
- *** AREA SUL ***
- *** SUDOESTE DA BAHIA ***
- *** AREA OESTE ***
- *** AREA TUCURUI-BELEM ***
- *** AREA MARANHAO ***
- ************* ITAIPU *************
- ************* COPEL **************



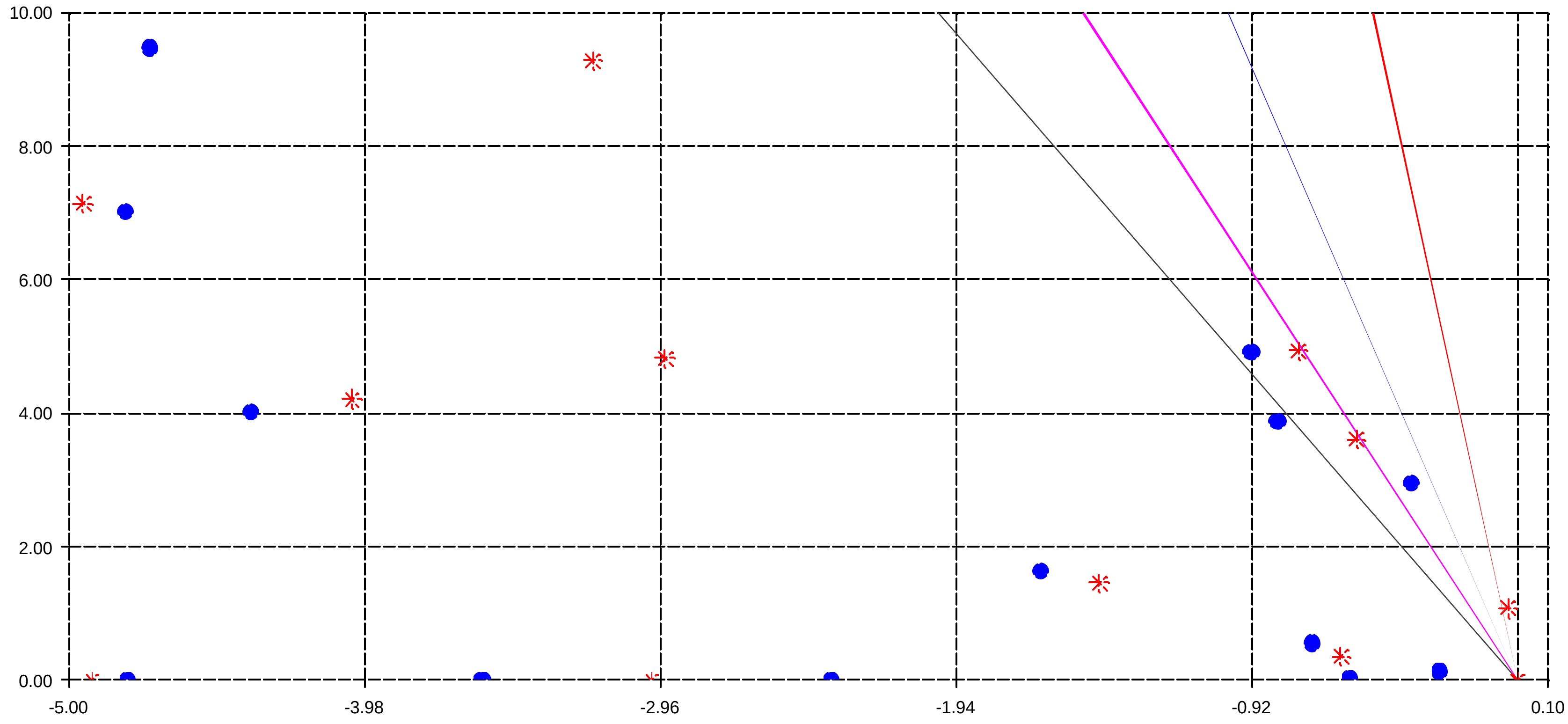
Pole (eigenvalue) Spectrum of $P_{ij}(s)$ / $B_{ij}(s)$



Pole-Zero Map of $P_{ij}(s)$ / $B_{ij}(s)$ (before cancellation)



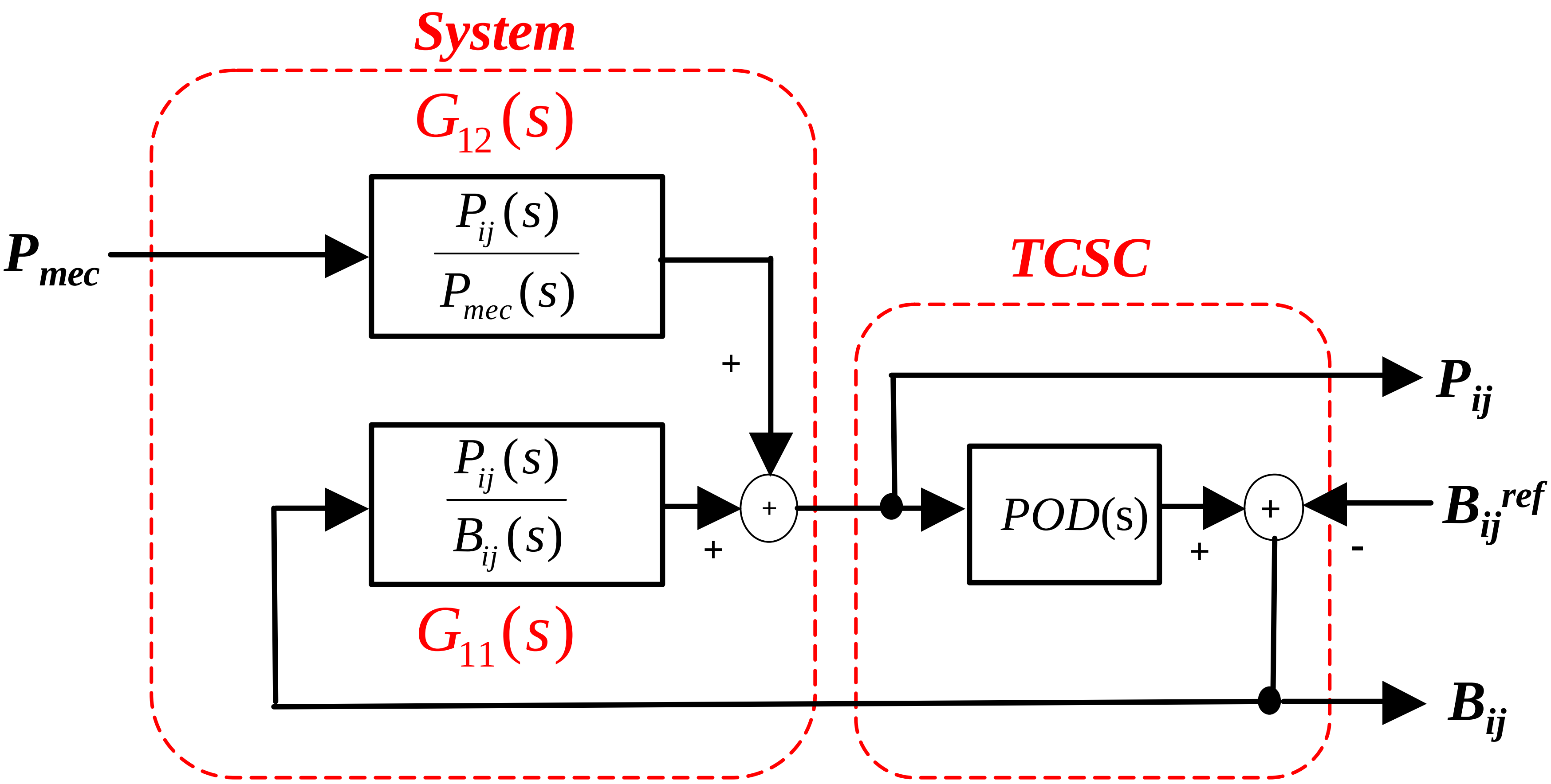
Pole-Zero Map of $P_{ij}(s) / B_{ij}(s)$ (after cancellation)



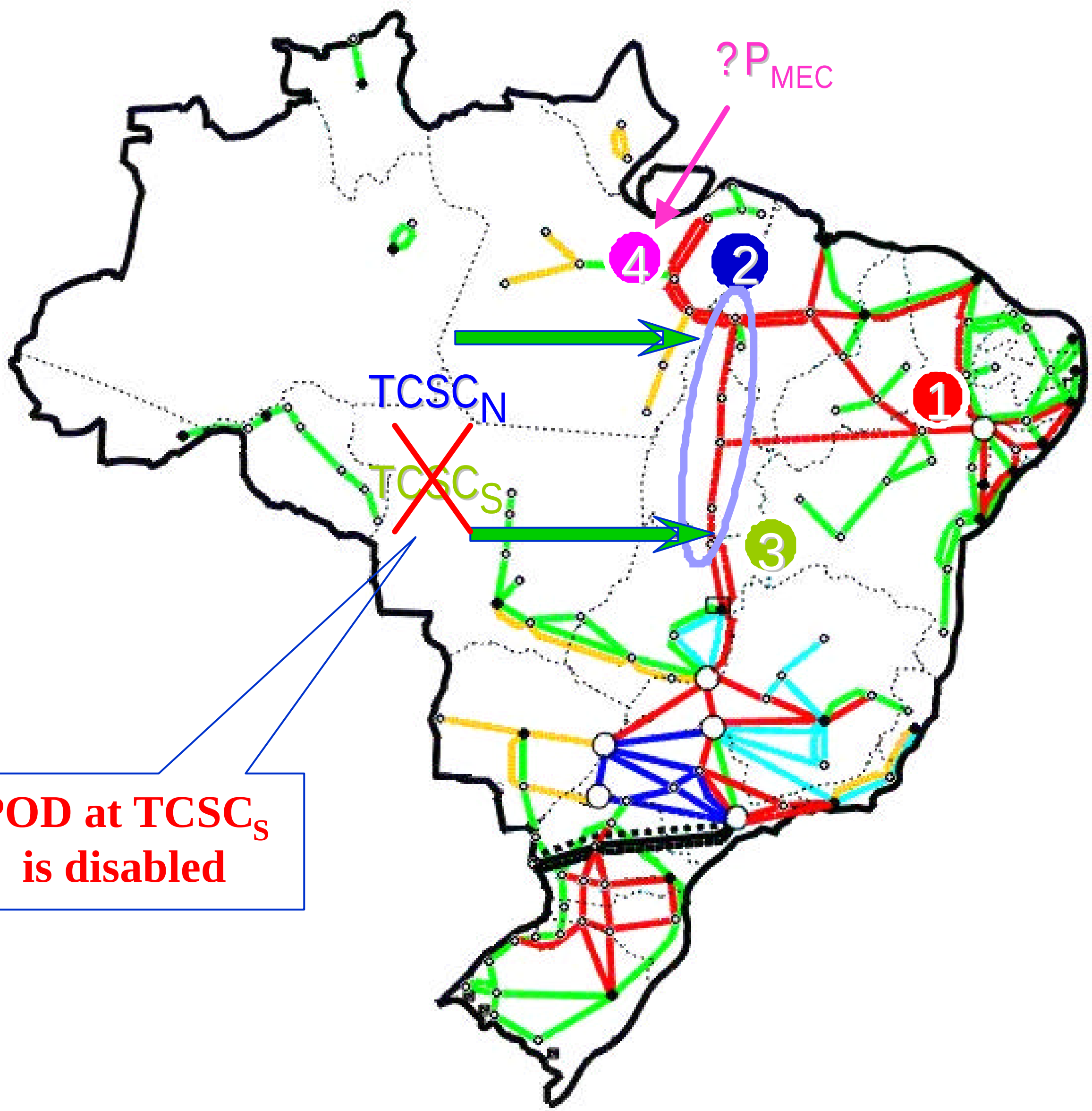
Transfer Functions of Interest

$G_{11}(s) \rightarrow$ POD loop of TCSC located at the NS Intertie

$G_{12}(s) \rightarrow$ External Disturbance applied at Tucuruí Power Plant



Brazilian North-South Interconnection – with one POD



POD at $TCSC_S$ is disabled

Power Plants/Substations

1 – Northeast Power Plants

2 – Imperatriz

3 – Serra da Mesa

4 - Tucuruí

Monitored Variables

? P_{ij} at North-South Tie

? B_{ij} at North-South Tie

Input Disturbance

? P_{MEC} at Tucuruí (Exogenous)

Dominant Poles and Associated $G_{11}(l)$ Residues (Reduced Models #1 and #2)

Num. Mode	Modes		Residues of $G_{11}(s)$		Model	
	Real	**Imaginary**	**Magnitude**	**Phase**	**#1**	**#2**
2	-2.9445	+4.8214j	0.0096215	+174.41	YES	YES
4	-3.1928	+9.2818j	0.0052552	+17.371	YES	YES
6	-2.6033	+10.722j	0.0037822	+84.793	YES	YES
8	-6.4231	+8.6949j	0.0018626	+108.62	YES	YES
10	-4.5154	+11.747j	0.0012366	+106.8	YES	YES
12	-0.033534	+1.0787j	0.00091216	+87.37	YES	YES
14	-4.9526	+7.1481j	0.00088318	+175.02	---	YES
16	-5.5632	+7.751j	0.00053183	-15.354	YES	YES
18	-0.75843	+4.9367j	0.00049972	+118.43	YES	YES
20	-1.4463	+1.4565j	0.00042096	-44.174	---	YES
22	-0.55674	+3.6097j	0.00039121	+109.89	YES	YES
24	-1.2786	+7.2546j	0.0003424	+155.51	---	YES
26	-1.2936	+1.4028j	0.00030419	-158.17	---	YES
28	-4.5467	+5.7598j	0.00020616	+89.349	---	YES
30	-1.2891	+8.5414j	+0.0001683	+154.87	---	YES
32	-0.61201	+0.35873j	0.00015938	-130.08	YES	YES
34	-0.11506	+0.23972j	0.000011953	+164.18	YES	YES

$$G_{11}(s) = \frac{P_{ij}(s)}{B_{ij}(s)}$$

Model #1 – order 22

Model #2 – order 34

Dominant Poles and Associated $G_{12}(l)$ Residues (Reduced Models #1 and #2)

Num. Mode	Modes		Residues of $G_{12}(s)$		Model	
	Real	Imaginary	Magnitude	Phase	#1	#2
2	-2.9445	+4.8214j	+2.0963	-89.569	YES	YES
4	-3.1928	+9.2818j	+0.51392	+171.22	YES	YES
6	-2.6033	+10.722j	+0.38095	-106.73	YES	YES
8	-6.4231	+8.6949j	+0.14588	-54.574	YES	YES
10	-4.5154	+11.747j	+0.14165	+89.765	YES	YES
12	-0.033534	+1.0787j	+1.4680	-109.36	YES	YES
14	-4.9526	+7.1481j	+0.073477	-64.545	---	YES
16	-5.5632	+7.751j	+0.033621	+108.47	YES	YES
18	-0.75843	+4.9367j	+0.23472	-86.159	YES	YES
20	-1.4463	+1.4565j	+1.9988	+11.244	---	YES
22	-0.55674	+3.6097j	+0.63001	-84.887	YES	YES
24	-1.2786	+7.2546j	+0.062051	-42.846	---	YES
26	-1.2936	+1.4028j	+1.1289	-108.72	---	YES
28	-4.5467	+5.7598j	+0.023311	-166.85	---	YES
30	-1.2891	+8.5414j	+0.093332	+109.3	---	YES
32	-0.61201	+0.35873j	+1.77430	+86.623	YES	YES
34	-0.11506	+0.23972j	+0.093455	+12.098	YES	YES

$$G_{12}(s) = \frac{P_{ij}(s)}{P_{mec}(s)}$$

Model #1 – order 22

Model #2 – order 34

Complex Diagonal Form and *Block-Diagonal State* Realizations with Related Similarity Transformation Matrix (fourth-order system example)

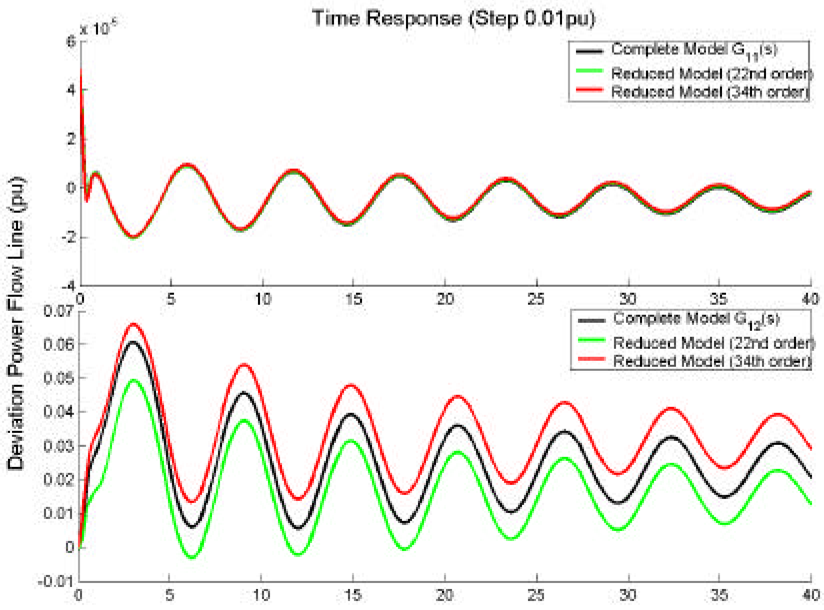
$$\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right] = \left[\begin{array}{cc:cc:c} l_1 & 0 & 0 & 0 & R_1 \\ 0 & l_1^* & 0 & 0 & R_1^* \\ \hdashline 0 & 0 & l_2 & 0 & R_2 \\ 0 & 0 & 0 & l_2^* & R_2^* \\ \hline 1 & 1 & 1 & 1 & d \end{array}\right]$$

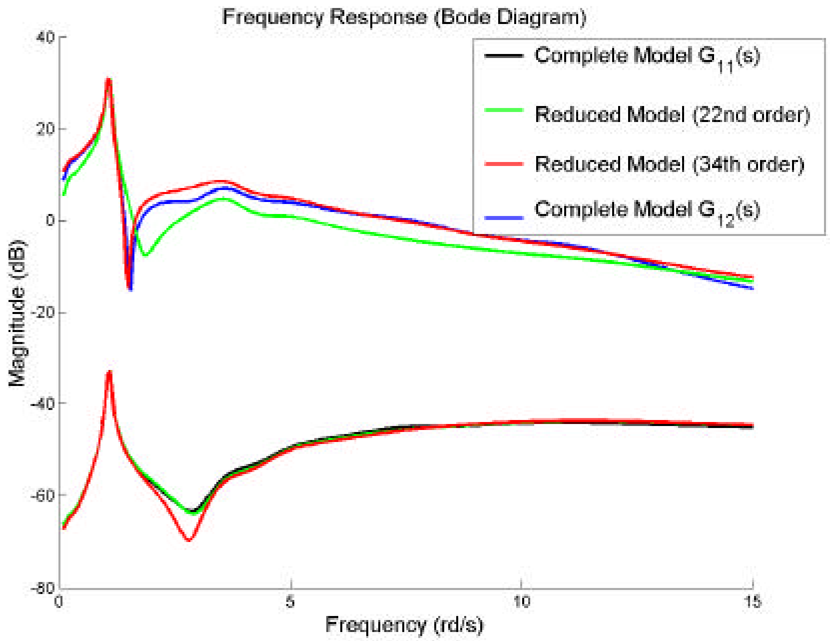
$$T = \left[\begin{array}{cc:cc} 1/\sqrt{2} & -j/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & j/\sqrt{2} & 0 & 0 \\ \hdashline 0 & 0 & 1/\sqrt{2} & -j/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & j/\sqrt{2} \end{array}\right]$$

$$\left[\begin{array}{c:c} A_2 & b_2 \\ \hdashline c_2 & d \end{array}\right] = \left[\begin{array}{c|c} T^{-1}AT & T^{-1}b \\ \hline c\,T & d \end{array}\right] = \left[\begin{array}{cc:cc:c} \mathrm{Re}(l_1) & \mathrm{Im}(l_1) & 0 & 0 & \mathrm{Re}(R_1)\sqrt{2} \\ -\mathrm{Im}(l_1) & \mathrm{Re}(l_1) & 0 & 0 & -\mathrm{Im}(R_1)\sqrt{2} \\ \hdashline 0 & 0 & \mathrm{Re}(l_2) & \mathrm{Im}(l_2) & \mathrm{Re}(R_2)\sqrt{2} \\ 0 & 0 & -\mathrm{Im}(l_2) & \mathrm{Re}(l_2) & -\mathrm{Im}(R_2)\sqrt{2} \\ \hdashline \sqrt{2} & 0 & \sqrt{2} & 0 & d \end{array}\right]$$

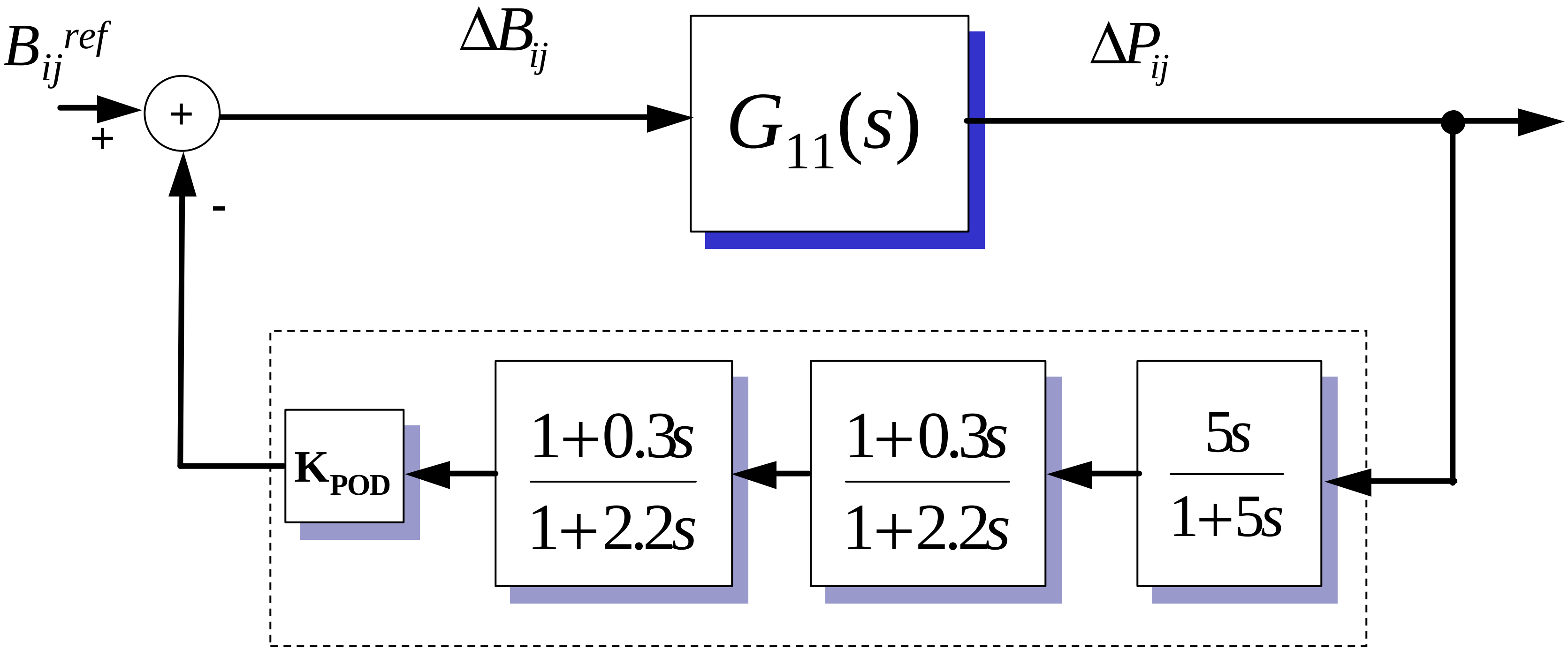
Time and Frequency Response Plots
Complete and Reduced Models

46 PSS but no POD controller (1,664 states)



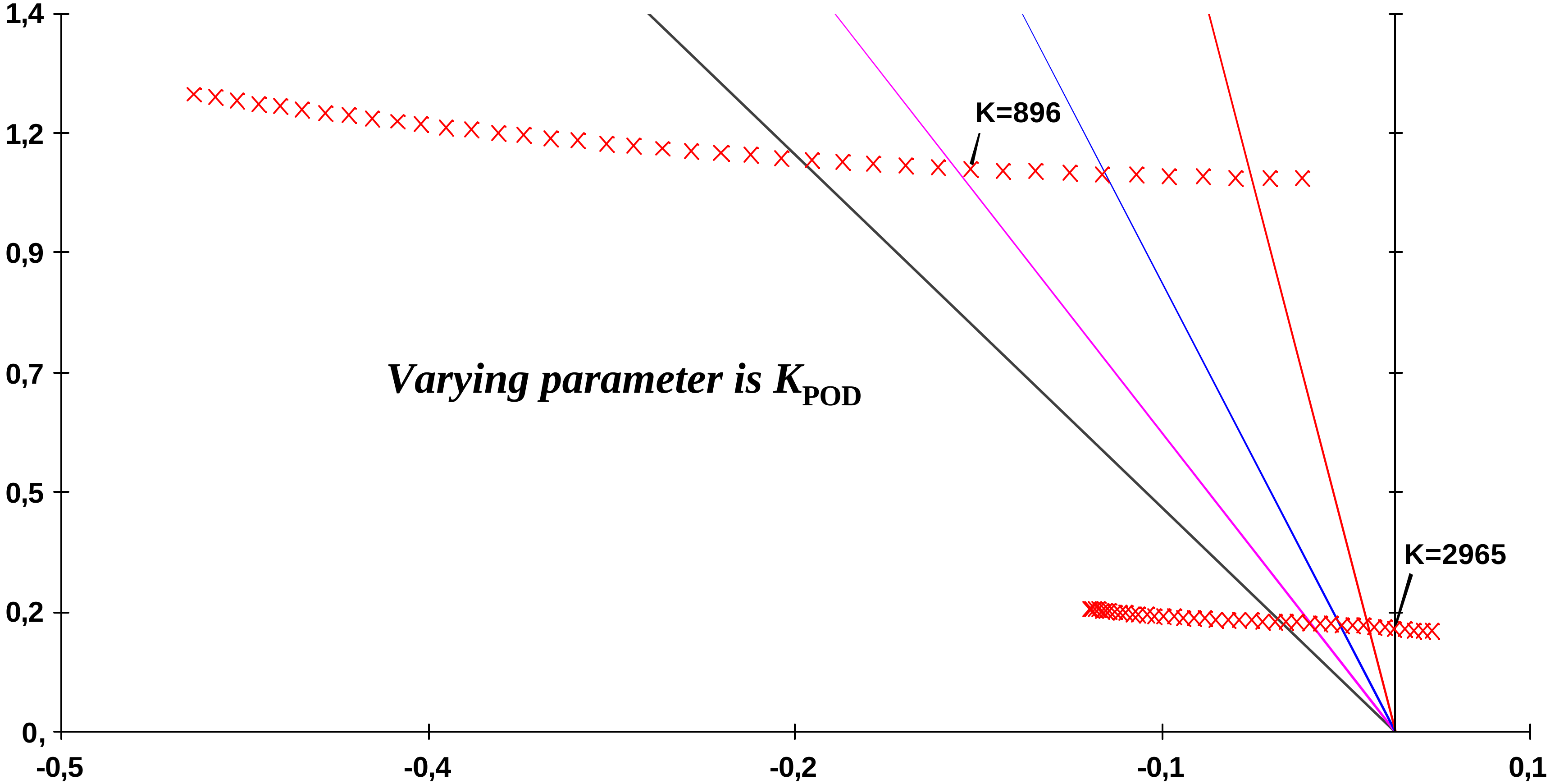


System with TCSC Equipped with POD Controller

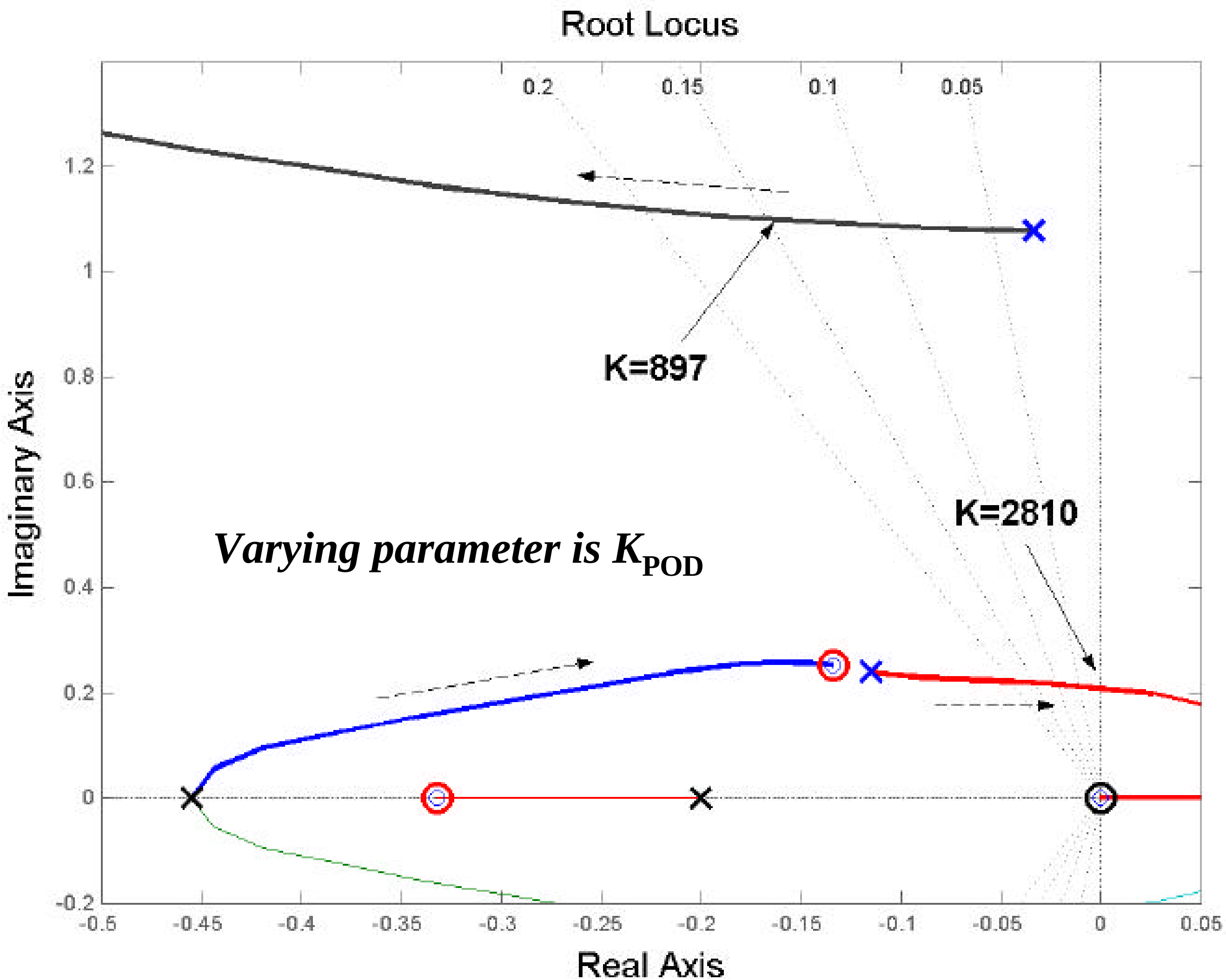


***POD* (s)**

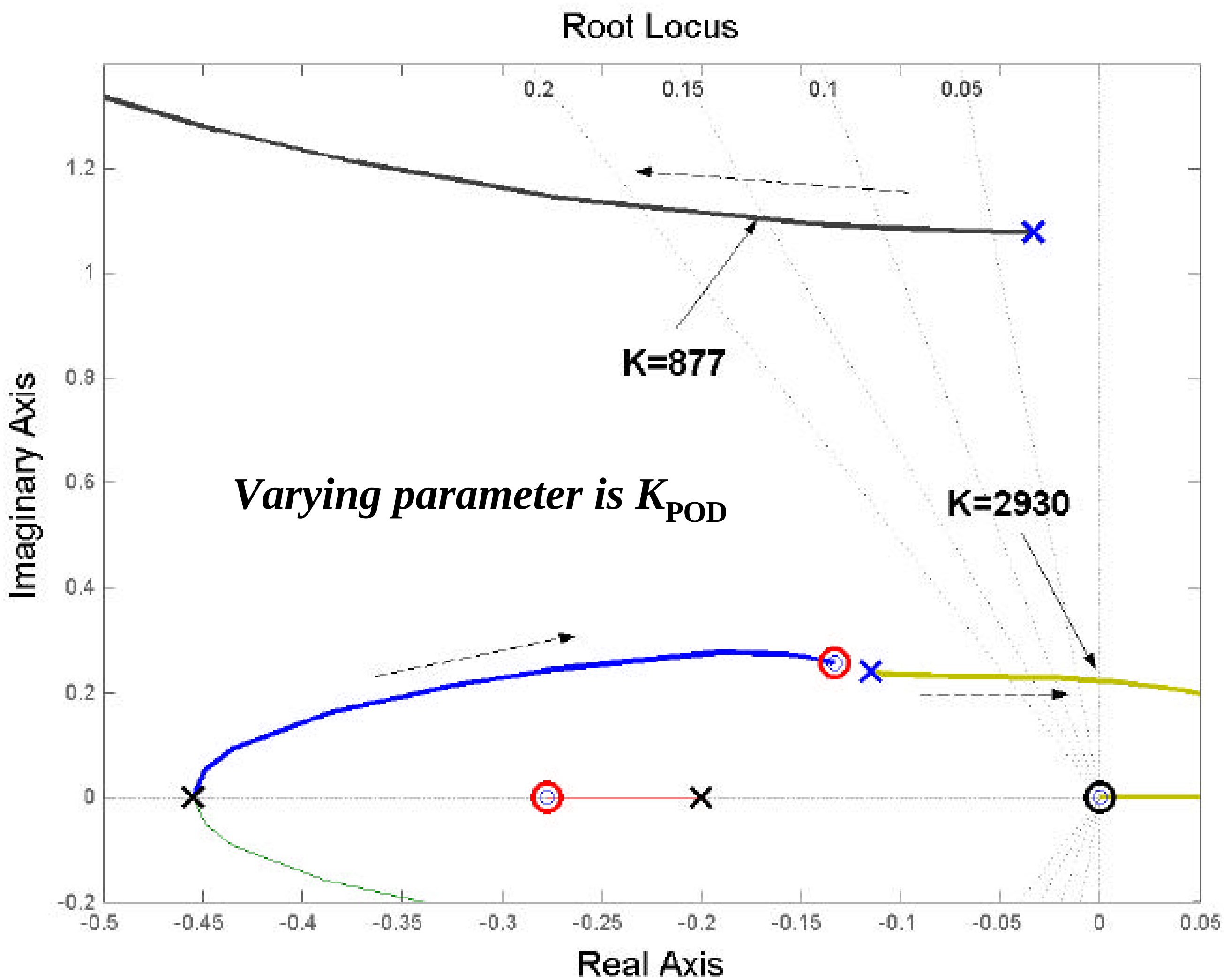
Root-Locus Branches for the Critical Poles Full-Order Model of $G_{11}(s)$ - 1667 states



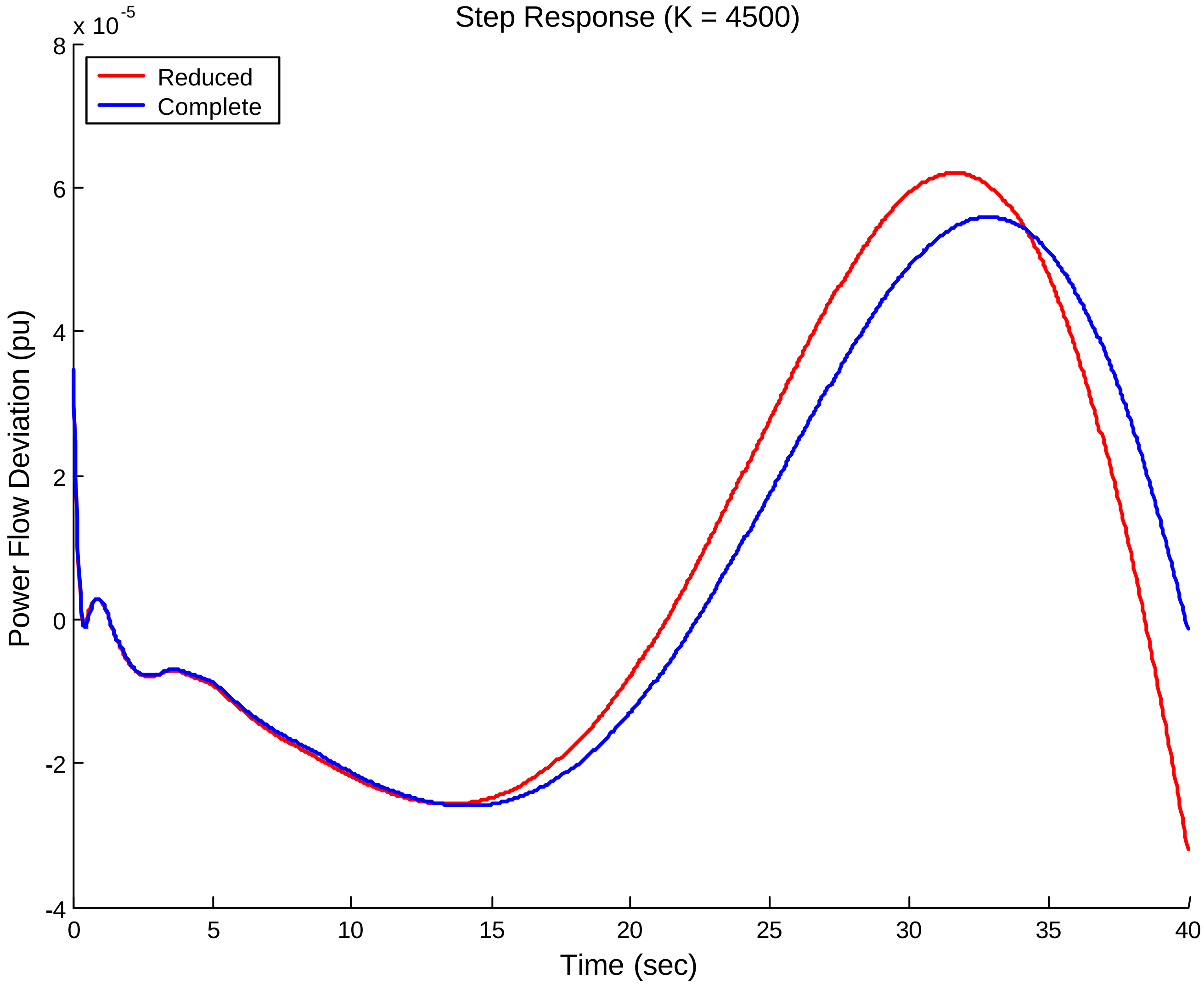
Root-Locus Branches for the Critical Poles 22th-Order Modal Equivalent of $G_{11}(s)$



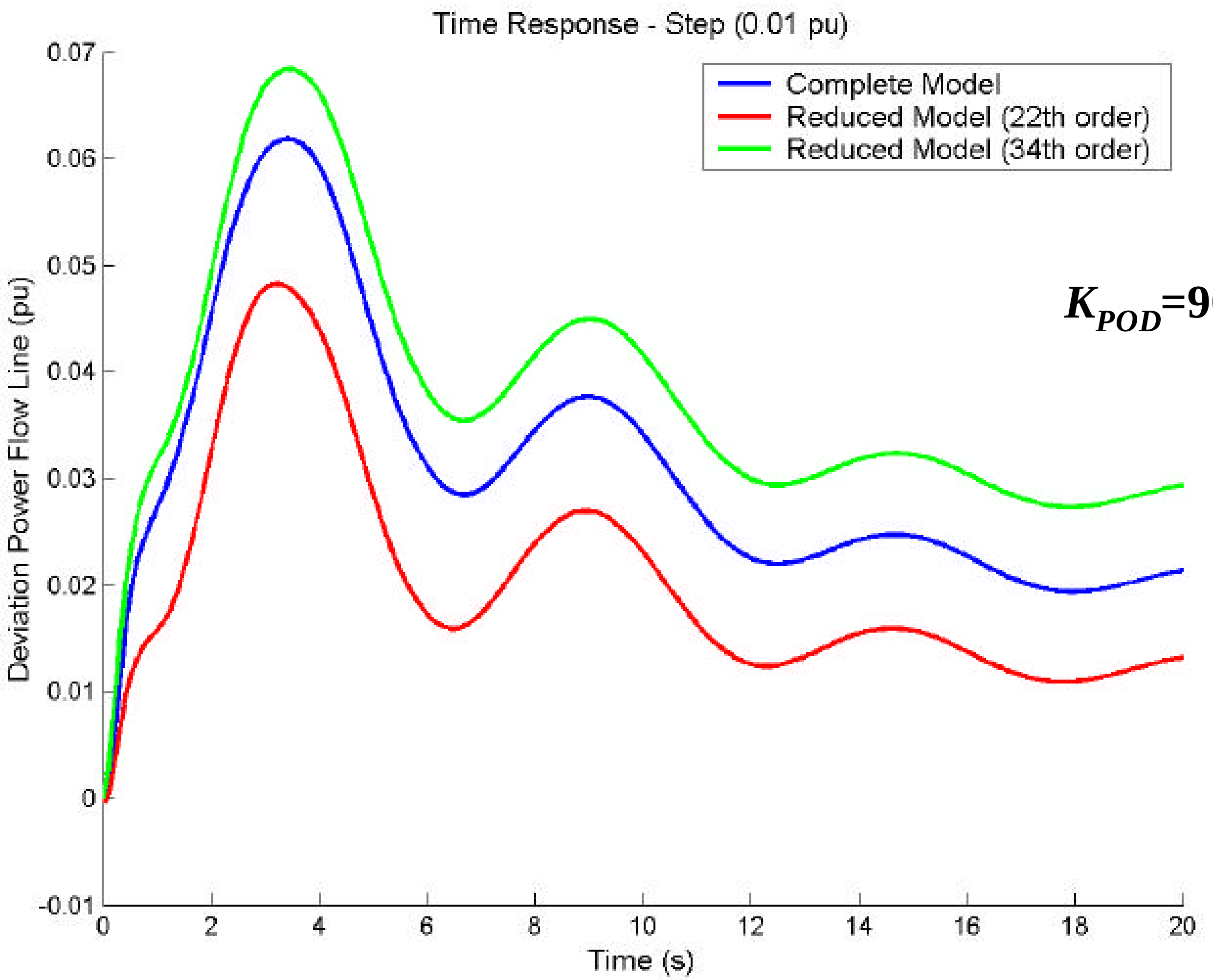
Root-Locus Branches for the Critical Poles 34th-Order Modal Equivalent of $G_{11}(s)$



Time Response Plots
Complete (1,664 states) and 34th-Order Models
POD Controller Gain varying from 900 to 4500 p.u.

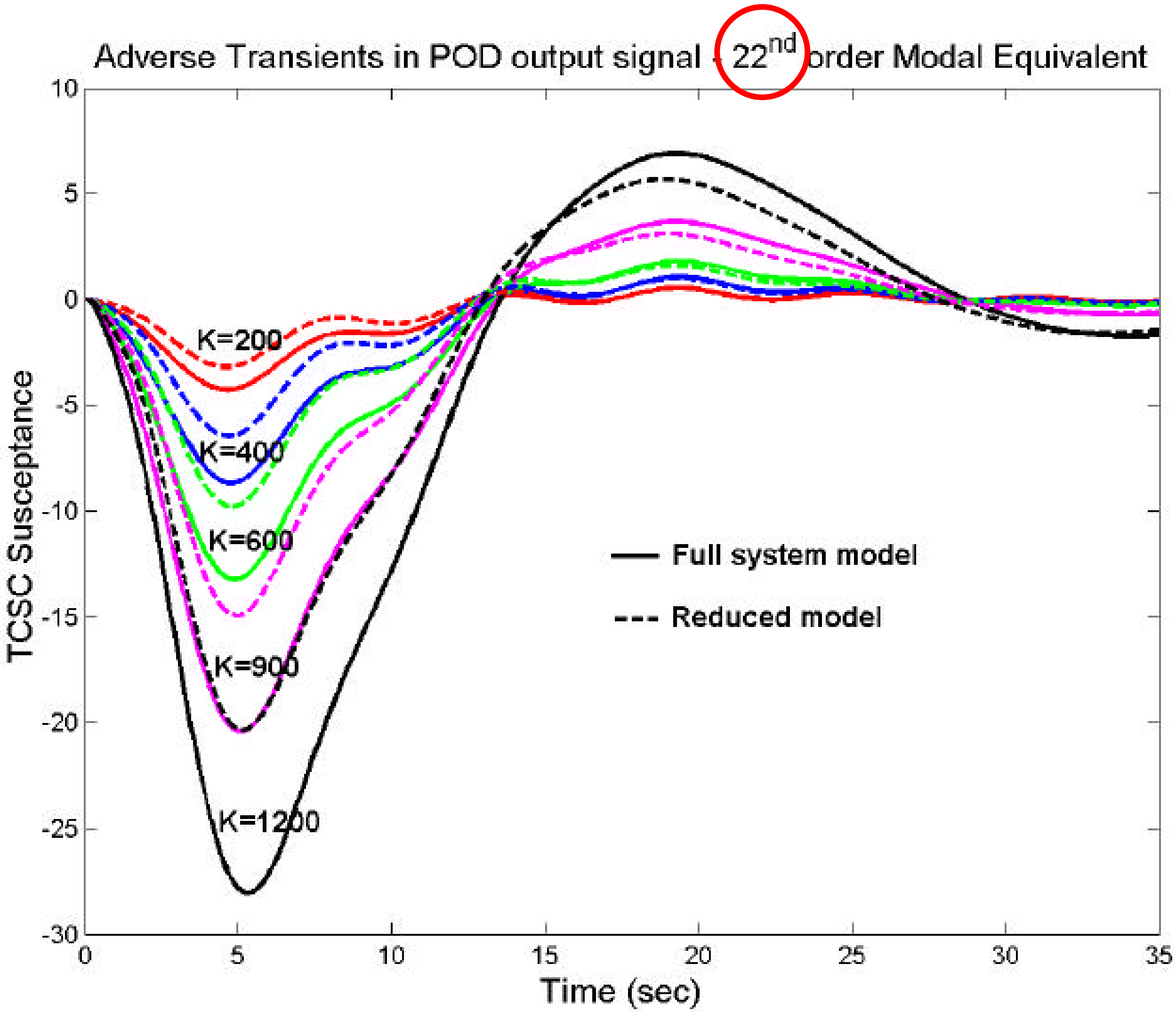


Step Responses for Closed-Loop System
Full System (1,667 states) and Modal Equivalents

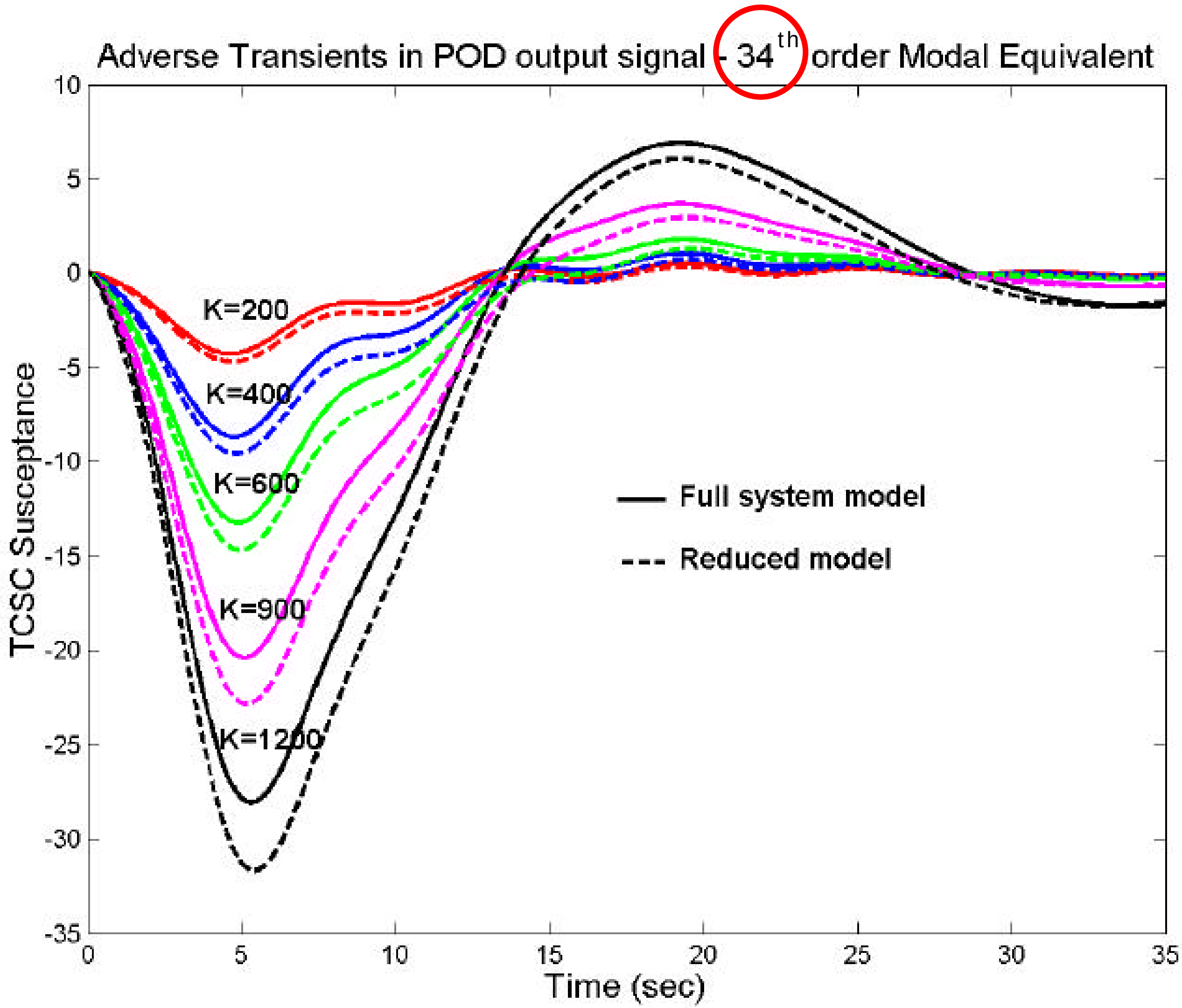


K_{POD}=900

Transients Appearing in POD Output Following Exogenous Disturbance, for Several POD gains



Transients Appearing in POD Output Following Exogenous Disturbance, for Several POD gains



Conclusions

- **Modal Equivalents**
  - Reduced models built with TF dominant poles and associated residues.
  - May be applied to large scale systems if efficient TF dominant pole routines are available.
  - TF zeros of the original model are not preserved.
  - Good reduced models are not of minimal order.

- **Balanced Truncation**
  - Good reduced models are of minimal order.
  - TF poles and zeros of the original model are not preserved.
  - Prohibitive computational cost for large systems.

Balanced Truncation applied to the 34th-order modal equivalent produced a good reduced model of 7th-order.