IX SEPOPE
23 a 27 de maio de 2004
May, 23th to 27rd – 2004
Rio de Janeiro (RJ) – Brasil

IX SIMPÓSIO DE ESPECIALISTAS EM PLANEJAMENTO
DA OPERAÇÃO E EXPANSÃO ELÉTRICA

IX SYMPOSIUM OF SPECIALISTS IN ELECTRIC
OPERATIONAL AND EXPANSION PLANNING

SP–110

Continuation Power Flow With the Help of the Cric Method

Fritz Walter Mohn [1,2] A. C. Zambroni de Souza [1] Nelson Martins [3]

1- Grupo de Engenharia de Sistemas - GESis, Escola Federal de Engenharia de Itajubá, BRAZIL
2- Agência Goiana de Regulação, Controle e Fiscalização de Serviços Públicos - AGR CEP 74 000-000, Goiânia, GO - BRAZIL
3- CEPEL – Centro de Pesquisas de Energia Elétrica, Ilha do Fundão , Rio de Janeiro, Brazil

ABSTRACT – This paper studies some decoupled techniques applied to continuation method. Because continuation method tends to provide accurate results and trace the bifurcation diagram, it tends to be appealling. The only restriction is the huge computational time required. In order to reduce this burden, each operating point is calculated according to the full Newton method, providing a benchmark for the other methodologies. These methodologies are based on the fast decoupled power flow, CRIC method and modified CRIC method. The idea is to study how fast and accurately the system load margin may be calculated with no loss of information regarding the critical buses. The tests are executed using some real power systems, and all the system limits are taken into consideration.

KEYWORDS: Voltage collapse, continuation methods, cric method.

I. INTRODUCTION On assessing a power system security, voltage collapse becomes a matter of concern, since it may be a source of serious problems. Most of the works in the literature employ a power flow model, which has driven many researchers to assess voltage collapse under a static point of view, even though the consequences of the phenomenon are dynamic [1].

Several papers have shown that the voltage collapse point may be associated with a saddle node bifurcation where a zero real eigenvalue is identified. Therefore, power system Jacobian becomes singular. Because no solution is obtained beyond this critical point, this commonly known as the maximum loadability point. It is also known, from the literature, that voltage collapse is a local or at most regional phenomenon [2]. Hence, identifying the system critical bus, i.e., the bus where the problem is originated, is also important. In this sense, continuation method [3-5] tends to provide accurate results regarding the load margin whereas identifying correctly the system critical buses. On the other hand, the computational time involved may be a barrier, since several operating points need to be evaluated in order to find the bifurcation point.

From the summary above, it can be concluded that reducing the computational time associated with continuation method could play an important role on voltage collapse analysis. That is the aim of this paper. For this purpose, a fast decoupled continuation method is proposed. With this purpose, fast decoupled-based continuation methods have been proposed [6, 7, 8], and good results have been reported. However, as a system is loaded, the decoupling may become weaker, and convergence problems are expected. Critical bus identification is also impacted [6]. As a consequence, this function requires the computation of the full load flow Jacobian.

In this paper, a continuation power flow is proposed based on the Cric method [9]. The CRIC method is also based on the decoupling of the set of the power flow equations, but the reactive power process takes into consideration the active power. The results obtained are encouraging, since the accuracy is kept, whereas the computational time is reduced and the critical buses identification is correctly obtained. The described tests were carried out for some Brazilian sample systems.

II. THE CRIC METHOD

Reference [6] proposes the use of fast continuation power flow in the continuation method. An improvement in the decoupled method is the CRIC method, presented by [9]. Assume the set of equations:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\underbrace{\begin{bmatrix}H & N\\ M & L\end{bmatrix}}_{J}\begin{bmatrix}\Delta q\\ \Delta V\end{bmatrix} \quad (1)$$

Hence:

$\Delta P = H\Delta\theta + N\Delta V$ (2)

$\Delta Q = M\Delta\theta + L\Delta V$ (3)

Equations (2) and (3) represent the active and reactive power cycles, respectively. The CRIC method is based on two principles:

- ✓ During the reactive power cycle, the power active injections are considered to be constant.
- ✓ Considering the assumption above is equivalent to assume the active power flows in the branches as constant.

For the active power cycle (2), the second term vanishes, since no voltage level variation is considered,

$\Delta\theta = H^{-1}\Delta P$ (4)

As for the reactive power cycle, one has:

$0 = H\Delta\theta + N\Delta V$ (5)

$\Delta Q = M\Delta\theta + L\Delta V$ (6)

Yielding:

$\Delta V = L'^{-1}\,\Delta Q$ (7)

where:

$L' = (L - MH^{-1}N)$ (8)

Note that equation (8) keeps the coupling during the reactive power cycle. However, matrix L' is not sparse, making the process infeasible for large power systems. The alternative is to obtain a sparse matrix that could present the same characteristics as presented by matrix L'. For this sake, the equations are rewritten in such a way that the active power flows in the branches are considered as constant:

$$P_{ik} = -aG_{ik}V_i^2 + Y_{ik}V_iV_k\cos(\theta_{ik} - \alpha) \quad (9)$$

$$Q_{ik} = aB_{ik}V_i^2 - V_i^2B_{sik} + Y_{ik}V_iV_k\,\text{sen}(\theta_{ik} - \alpha) \quad (10)$$

If P_{ik} is constant, the following expression arises:

$$Q_i = \sum_{k=1\neq i}^{n}\left(a\,B_{ik}V_i^2 - V_i^2B_{sik} + \sqrt{(Y_{ik}V_iV_k)^2 - \left(P_{ik} + a\,G_{ik}V_i^2\right)^2}\right) \quad (11)$$

From equation above, the CRIC Jacobian and the sensitivities $\partial Q/\partial V$ are obtained, whereas the sparsity is kept.

$$LC_{ii} = \frac{\partial Q_i}{\partial V_i}$$

$$LC_{ik} = \frac{\partial Q_i}{\partial V_k}$$

$$\frac{\partial Q_i}{\partial V_i} = \sum_{k=1\neq i}^{n}\left(2aB_{ik}V_i - 2V_iB_{sik} + \frac{Y_{ik}^2V_iV_k^2 - \left(2aG_{ik}V_iP_{ik} + 2a^2G_{ik}^2V_i^3\right)}{Y_{ik}V_iV_k\,\text{sen}(\theta_{ik} - \alpha)}\right) \quad (12)$$

$$\frac{\partial Q_i}{\partial V_k} = \frac{Y_{ik}V_i}{\text{sen}(\theta_{ik} - \alpha)} \quad (13)$$

where:

a : transformer tap.

B_{sk} : transmission line susceptance.

G_{ik}, B_{ik} e Y_{ik}: elements of admittance of branch ik.

α: angle between B and G.

III. The Continuation Method

Continuation methods may be used to trace the path of a power system from a stable equilibrium point up to a bifurcation point [3]. Such a methodology is based on the following system model:

$f(x, \lambda) = 0$ (14)

where x represents the state variables and λ is a system parameter, used to drive a system from one equilibrium point to another. This type of model has been employed for numerous voltage collapse studies, with λ been considered as the system load/generation increase factor or power transfer level. Two steps move the system along the bifurcation path:

1- Predictor step, which defines a direction for load and generation increase. Tangent vector may be used for this purpose, and is given by:

$$TV = \begin{bmatrix} D\theta \\ DV \end{bmatrix}\frac{1}{\Delta\lambda} = J^{-1}\begin{bmatrix} P_o \\ Q_o \end{bmatrix} \quad (15)$$

where J is the load flow Jacobian, θ and V the state variables (angle phase and voltage magnitude, respectively), and P_o and Q_o are net active and reactive powers connected to each bus. *TV* is the acronysm for tangent vector. The predictor step is then given by:

$$\Delta\lambda = 1/\|TV\|$$

where $\|.\|$ stands for tangent vector norm. From this expression, the steeper the curve, the smaller the predictor step, and vice versa. The method takes bigger steps when the system is far away from the bifurcation point and smaller steps as the bifurcation is approached. The actual operating point is obtained with the help of the corrector stage.

2- Corrector step, obtained by the inclusion of an extra equation. Such an equation comes from the fact that the predictor and corrector vectors are perpendicular to each other.

The methodology above describes the continuation method in a general sense. The structure of the Jacobian matrix associated with this method is given by:

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ rsh1 \end{bmatrix} = \begin{bmatrix} H & N & k1 \\ M & L & \\ FF1 & & kF1 \end{bmatrix} \times \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta\lambda \end{bmatrix} \quad (16)$$

Where, H, N, M and L stand for the power flow Jacobian submatrices, and vector $K1$ represents the predetermined generation and load increase direction. $FF1$ and $KF1$ correspond to parameterization equation in the continuation power flow. During the "normal" region of continuation, $kF1$ equals one and $FF1$ is a zero row.

IV. CONTINUATION METHOD AND CRIC

Applying CRIC to the continuation method follows the same approach as seen in [6], when the fast decoupled method is used. The aim of the method is to calculate the load margin and identify the critical buses. Besides this output, a special study about the singularity of the matrix LC used in the CRIC method is also carried out. In order to speed up the calculation process, an alternative is proposed, which consists of keeping the matrix LC constant for each equilibrium point during the bifurcation path determination. This is called fast CRIC. The characteristics considered in the CRIC continuation method are described next:

Predictor Step Size

It is obtained, in the classical method, with the help of tangent vector. Such a vector is also used here, but it is employed in relation to the matrix LC, as shown next:

$$TV_{LC} = [\Delta V / \Delta I] = LC^{-1} Q_o \tag{17}$$

Index of Collapse (IC)

Reference [10] analyzes in detail a set of voltage collapse indices. In particular, it shows that tangent vector converges to the center manifold, which enables one to identify the vanishing eigenvalue as:

$$IC = TV^t * J * TV \tag{18}$$

where : TV ➔ Tangent Vector

J ➔ Jacobian

As the system approaches the bifurcation, IC tends to zero quite rapidly. The IC index may therefore be used to stop the calculation with no loss of accuracy, since the continuation method calculates a large number of operating points close to the bifurcation.

Identification of Critical Buses

As already proposed in the literature, tangent vector may be used as a tool to identify the system critical buses [10]. This is based on the fact that its entries provide the sensitivity of the state variables as a function of the system parameter. In this paper, this identification is also investigated when tangent vector is obtained from matrix LC, and the results will be compared with the ones obtained from the complete load flow Jacobian.

V. RESULTS

Four practical systems are used to test the ideas proposed in the paper. These systems contain 214, 412, 721 and 1381 buses, respectively, and all of them are studied considering their true operating limits. For each system, the load margin is calculated with the help of the continuation method. Each equilibrium point is obtained by the Full Newton method (FN), Fast decoupled load flow (FD), CRIC (NC) and Fast CRIC (FC).

At this part of the test, tangent vector is used according to the complete load flow Jacobian . Hence, for each equilibrium point, the full Jacobian is calculated, and tangent vector is derived. It ensures the same step size for all the methods. The stopping criterion is not used at this time. Table I presents the results obtained.

Table I – Load Margin and Computational Time (Case 1)

System	**Load Margin (pu)**				**Time (pu)**			
	FN	**FD**	**NC**	**FC**	**FN**	**FD**	**NC**	**FC**
214	1,326	1,326	1,291	1,300	1,00	1,032	0,983	0,763
412	1,115	1,116	1,115	1,115	1,00	0,650	0,910	0,631
721	1,061	1,060	1,060	1,060	1,00	0,830	0,927	0,789
1381	1,119	**DIV**	1,118	1,117	1,00	**DIV**	0,831	0,703

It is important to note that because the computational time may vary as a function of the computer used, it is measured here in relation to the time observed when the Full Newton method is used. From now on, this will be the reference. Note that the system with 1381 buses presents divergence problems when the Fast Decoupled method is used. This system contains several transmission lines with high ratio R/X.

The tests presented in Table I are executed again. This time, the stopping criterion is used. As for the decoupled methods, when the number of iterations become high or divergence problems occur, the method is changed to the Full Newton.

Table II –Load Margin and Computational Time (Case 2)

System	**Load Margin (pu)**				**Time (pu)**			
	FN	**FD**	**NC**	**FC**	**FN**	**FD**	**NC**	**FC**
721	1,060	1,060	1,060	1,060	0,871	0,767	0,812	0,739
1049	1,160	1,161	1,161	1,161	0,648	0,474	0,552	0,530
1381	1,106	1,106	1,106	1,106	0,714	0,740	0,626	0,596
1900	1,048	1,048	1,048	1,048	0,659	0,662	0,672	0,568

It is important to mention that the computational time shown in Table I is obtained as a function of the Full Newton method observed in Table I. This time, because the decoupled method may change to the Full Newton, the fast decoupled method does not fail, and the system with 1381 buses is correctly analyzed.

Besides the assumptions considered for the Case 2, the fact that the decoupled methods may converge easily for operating points far from the bifurcation point may be explored. For the full Newton method, the load factor variation is given by:

$$\Delta l = k \frac{1}{\|TV\|} \qquad (19)$$

where K is a constant. For the fast decoupled method, one has:

$$\text{TV} = \begin{bmatrix} \text{ÄV} \\ \text{Äë} \end{bmatrix} = B^{-1} Q_o V \qquad (20)$$

As for the CRIC method, tangent vector is given by equation (17). Calculating tangent vector according to the expressions above yields different step sizes for each method. The results obtained considering the assumptions already considered in the Case 2 with different step sizes are shown in Table III.

Table III – Load Margin and Computational Time (Case 3)

System	Load Margin (pu)				Time (pu)			
	FN	**FD**	**NC**	**FC**	**FN**	**FD**	**NC**	**FC**
721	1,060	1,060	1,060	1,060	0,871	0,414	0,525	0,545
1049	1,160	1,191	1,191	1,161	0,648	0,184	0,257	0,166
1381	1,106	1,106	1,108	1,108	0,714	0,730	0,176	0,233
1900	1,048	1,047	1,050	1,049	0,659	0,392	0,256	0,243

From Table III one can see that the computational time tremendously reduced, whereas the accuracy in the results is not compromised.

Classification of the Critical Buses

Table IV shows the 5 most critical buses for the test systems. Such a classification is carried out with the help of the complete Jacobian and the CRIC matrix.

Table IV – Critical Buses Classification for J and CRIC

System	Matrix	1	2	3	4	5
721	J	119	114	109	478	477
	LC	470	478	109	463	458
1049	J	308	309	311	756	312
	LC	308	309	311	312	756
1381	J	1357	1356	1246	1245	1247
	LC	1356	1355	1245	1244	1052
1900	J	1357	1352	1367	1298	1300
	LC	1357	1359	1367	1342	1298

Observe that the worst ranking applying tangent vector with matrix CRIC went to the system of 721 buses.

If that classification obtained with tangent vector applying Jacobiana full to consider only the relative components to the module of the voltage, the classification becomes similar:

Table V – Critical Buses ranking for J and CRIC – Just Voltage Level in J

System	Matrix	1	2	3	4	5
721	**J**	109	478	477	476	470
	LC	470	478	109	463	458

VI. CONCLUSION

This paper addressed the problem of load margin calculation when the CRIC method is incorporated into the continuation power flow. The characteristics of the CRIC method are described and some important aspects related to voltage collapse are exploited. The results obtained with the help of some real systems show that the method may be effective for load margin calculation, since the computational time is reduced and the accuracy is preserved.

Besides the reduction in the computational effort, the method showed to be robust, since no divergence problem was reported when some features incorporated to the method were considered. The system operating limits were fully considered in all tests. The authors believe there is room for further improvements and will continue this investigation

VII. REFERENCES

1 Walter J. Causarano, Djalma M. Falcao and Glauco N. Taranto, "A Fast Domain Simulation Method for Voltage Stability Assessment", *VI SEPOPE,* Transmission Cost and Power System Operation session, Paper SP-03, Salvador, Brazil, May 24-29, 1998. Available in CD.

2 Yakout Mansour, " Industry Parctice in voltage Stability Analysis of Power Systems,", *L. H. Fink Editor, Proc. Bulk Power System Voltage Phenomena III – Voltage Stability and Security,* ECC Inc., Davos Switzerland, 1994.

3 Claudio A. Cañizares, A. C. Zambroni de Souza, V. H. Quintana, "Comparison of Performance Indices for Detection of Proximity to Voltage Collapse," *IEEE Transactions on Power Systems,* Vol. 11, No. 3, August 1996, pp. 1441-1450.

4 C. A. Cañizares and F. L. Alvarado, "Point of Collapse and continuation methods for large ac/dc systems,"*IEEE Trans. Power Systems,*vol. 8, no. 1, February 1993, pp. 1-8.

5 V. Ajjarapu, "Identification of steady state voltage stability in power systems," *Proc. of International Conference on High Technology in the Power Industry,* March 1988, pp. 244-247.

6 Fritz Walter Mohn, A. C. Zambroni de Souza, "On Fast Decoupled Continuation Power Flows", *Electric Power Systems Research 63 (2002) 105/ 111*

7 Alves D. A., L. C. P. da Silva, C. A. Castro e V. F. da Costa., "Continuation Fast Decoupled Power Flow with Secant Predictor", *accepted for publication by IEEE Transactions on Power Systems.*

8 D. A. Alves, L. C. P. da Silva, C. A. Castro, and V. F. da Costa, "Parameterized fast decoupled load flows for tracing the Power Systems Bifurcation Diagrams," *1999 Proc. IEEE PES Summer Meeting*, pp. 708-713.

9 Carpentier, J.L., "CRIC: A New Active Reactive Decoupling Process in Load Flows, Optimal power Flows and System Control", *IFAC Symposium on Power Systems and Power Plant Control*, Beijing, 1986.

10 A C. Zambroni de Souza, "Discussion On Some Voltage Collapse Indices", *Electric Power Systems Research*, Vol. 53, No. 1, pp. 53-58, January 2000.