

# **STUDYING HARMONIC PROBLEMS USING A DESCRIPTOR SYSTEM APPROACH**

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# Background



- ↓ Harmonic voltage performance of a system depends on the location of its poles and zeros mainly with respect to the characteristic harmonic frequencies
- ↓ Knowledge of the poles, zeros and their respective sensitivities to system parameters
- ↓ Identification of changes in the system which will reduce harmonic voltage levels

# Difficulties



- ↓ The construction of the state matrix for practical systems is not a simple task.
- ↓ Methods based on state matrix formulations present some limitations regarding network topology and not automatically deal with state variables redundancy

# Descriptor System Approach



$$\mathbf{T} \dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}$$

$$\mathbf{y} = \mathbf{C} \mathbf{x}$$

- ↓ Overcomes the computational difficulties associated with the state matrix method.
- ↓ Automatically deals with state variable redundancies
- ↓ Can be efficiently applied to large-scale networks of any topology

# Objectives



- ↓ Application of the Descriptor System  
Approach to study harmonic problems

# Results

## ↓ Network Model

$$\left[ \begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array} \right] \cdot \left[ \begin{array}{c} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{v}}_{nodal} \end{array} \right] = \left[ \begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array} \right] \cdot \left[ \begin{array}{c} \mathbf{x}_1 \\ \mathbf{v}_{nodal} \end{array} \right] + \left[ \begin{array}{c} \mathbf{0} \\ \mathbf{I} \end{array} \right] \cdot \mathbf{i}_{nodal}$$

$$\mathbf{v}_{nodal} = \left[ \mathbf{0}^T \mid \mathbf{I} \right] \cdot \left[ \begin{array}{c} \mathbf{x}_1 \\ \mathbf{v}_{nodal} \end{array} \right]$$

## ↓ Impedance as a transfer function

$$\mathbf{Z} = \mathbf{C} \cdot (s \cdot \mathbf{T} - \mathbf{A})^{-1} \cdot \mathbf{B}$$

# Harmonic Impedance

$$Z_{kk} = \text{diag} \left[ (s \cdot \mathbf{T} - \mathbf{A})^{-1} \right]_{(2 \cdot n_l + k)} = \frac{\det(s \cdot \mathbf{T}_k - \mathbf{A}_k)}{\det(s \cdot \mathbf{T} - \mathbf{A})}$$

↓ The system poles correspond to the generalized eigenvalue problem associated with the matrix pair  $\{\mathbf{A}, \mathbf{T}\}$

$$\det(s \cdot \mathbf{T} - \mathbf{A}) = 0 \Leftrightarrow \mathbf{A} \cdot \mathbf{v}_i = \lambda_i \cdot \mathbf{T} \cdot \mathbf{v}_i$$

↓ The zeros, associated with the self impedance of node  $k$ , correspond to the generalized eigenvalue problem associated with the matrix pair  $\{\mathbf{A}_k, \mathbf{T}_k\}$

# Sensitivity Analysis

## ↓ Sensitivity

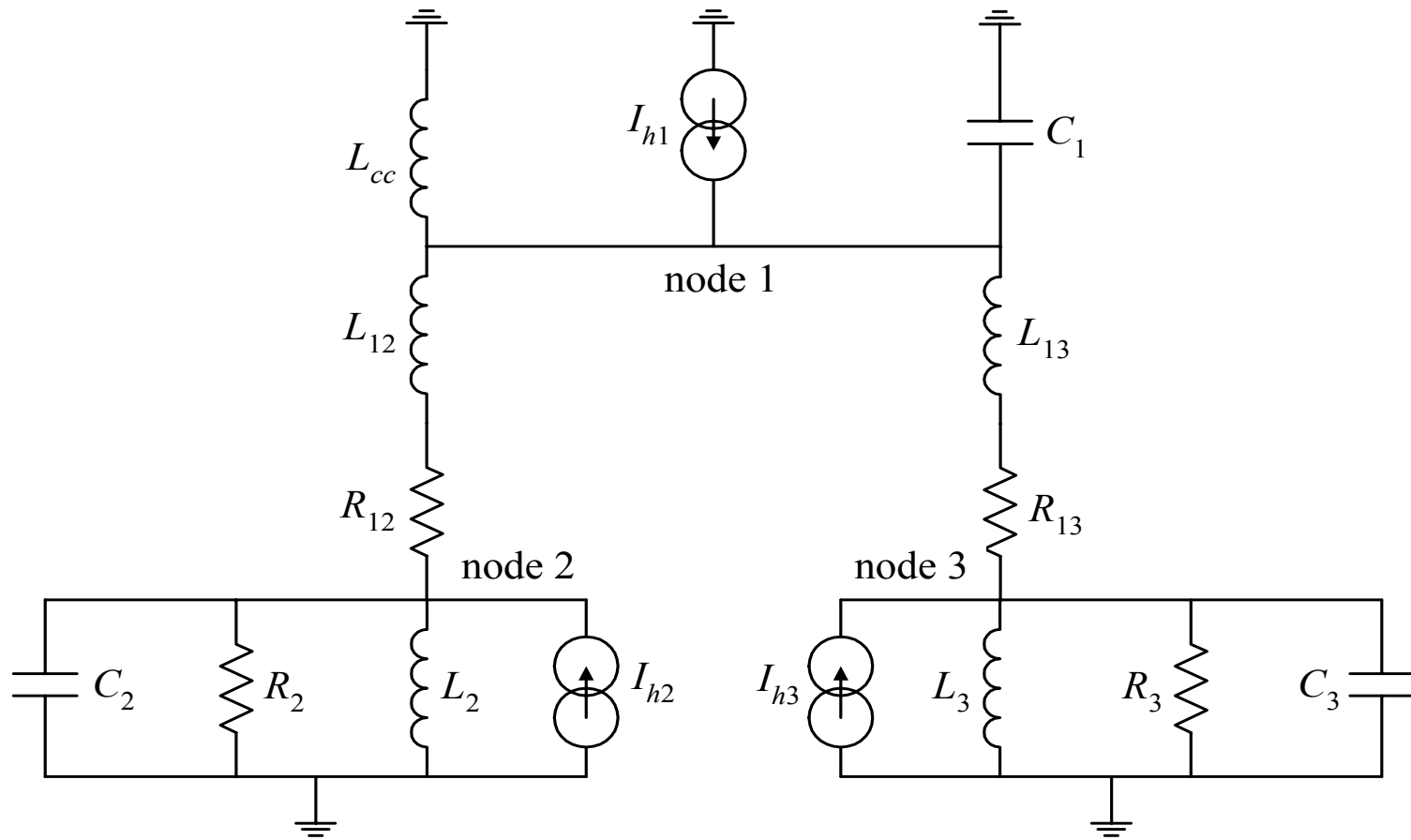
$$\frac{\partial \lambda_i}{\partial \alpha_j} = \frac{\mathbf{w}_i \cdot \left( \frac{\partial \mathbf{A}}{\partial \alpha_j} - \lambda_i \cdot \frac{\partial \mathbf{T}}{\partial \alpha_j} \right) \cdot \mathbf{v}_i}{\mathbf{w}_i \cdot \mathbf{T} \cdot \mathbf{v}_i}$$

## ↓ Eigenvalue Variation

$$\Delta \lambda_i = \alpha_j^0 \cdot \frac{\partial \lambda_i}{\partial \alpha_j} \left( \alpha_j^0 \right) \cdot \frac{\Delta \alpha_j}{\alpha_j^0}$$



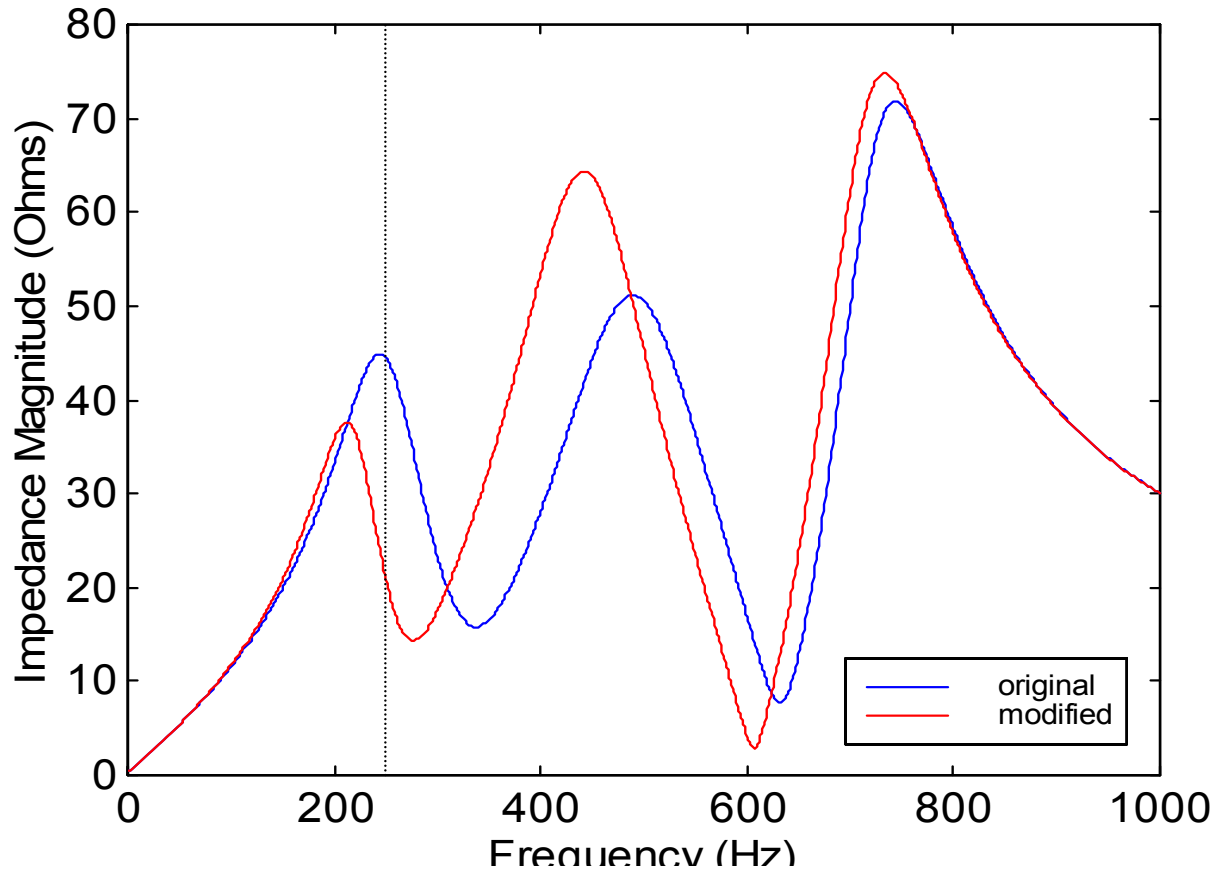
# Test System



# Poles and Zeros Sensitivities

	Poles			Zeros					
				Node 1		Node 2		Node 3	
	1	2	3	1	2	1	2	1	2
$f(\text{Hz})$	252	489	722	425	565	332	633	382	704
$L_{cc}$	-633	-68	-312	0	0	-302	-501	-551	-292
$L_2$	-18	-22	-11	0.0	-41	0	0	-33	-15
$L_3$	-22	-12	-1	-30	0	-29	-5	0	0
$L_{12}$	-11	-493	-1551	0	-1820	-248	-415	-232	-1819
$L_{13}$	-119	-949	-492	-1325	0	-470	-1080	-368	-199
$C_1$	-284	-74	-1295	0	0	-237	-1523	-533	-1188
$C_2$	-158	-732	-697	0	-1689	0	0	-685	-912
$C_3$	-339	-719	-178	-1317	0	-799	-452	0	0

# Shifting Poles and Zeros



$$C_{3\text{original}} = 11.9 \mu F$$

$$C_{3\text{new}} = 19.56 \mu F$$

Reduction of 47% in  
the impedance  
magnitude at 250 Hz

# Conclusions

- ↓ State-space based formulations obtain the same frequency domain results as the conventional methods based on nodal formulation and more:
  - ↙ Identification of elements mostly involved in specific resonances
  - ↙ Determination of the necessary changes in system elements in order to shift the location of poles and/or zeros to desired positions
  - ↙ Optimum allocation of capacitor banks and/or passive filters

# Conclusions



- ↓ Descriptor system approach allows:
  - ↙ Simple and efficient computational implementation
  - ↙ Ability to model systems of any topology and containing state variable redundancies
  - ↙ Applicability to large-scale networks, due to the very sparse matrices involved and the availability of powerful sparse eigensolution algorithms applied to descriptor systems

# Sparse Structure of Matrix A

